Knowledge-Based Systems 243 (2022) 108497

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Linguistic patterns as a framework for an expert knowledge representation in agent movement simulation

Jerzy Grobelny, Rafał Michalski *

Faculty of Management, Wrocław University of Science and Technology, Poland

ARTICLE INFO

Article history:
Received 13 November 2021
Received in revised form 22 February 2022
Accepted 23 February 2022
Available online 3 March 2022

Keywords:
Knowledge representation
Linguistic patterns
Multivalued logic
Fuzzy sets
Agent-based models
Social system dynamics

ABSTRACT

The article shows how the concept of *linguistic patterns* (LP) combined with expert knowledge can be used to model the dynamics of social groups. Linguistic expressions similar to natural language are the basis for generating virtual forces (VF) that govern the movements of the individual agent. Using logical sentences and providing the methodology to determine the degree of their truth led to the development of rules of agent behavior. Such an approach allows for constructing flexible simulation models. This paper describes this new idea in detail and illustrates its capabilities and properties by simple examples. Furthermore, a series of simulation experiments involving problems of known solutions with stable final configurations is performed and qualitatively analyzed. For additional validation, a suboptimal scattered plot generation method proposed by Drezner is used. A set of simulations for classic problems showed the convergence of agent dynamic behavior in the proposed framework with the solutions provided by the Drezner approach.

1. Introduction

Agent movement simulations are used to model complex systems and observe their overall behavior based on local relations and interactions of their components. Such a way of modeling has a rich tradition and has been widely applied to explore the dynamics of social as well as abstract systems. Studying segregation problems has greatly contributed to the development of this trend.

This current article presents a novel conceptual framework for a study of agent movement within relatively small social groups. In general, the fundamental objective of this concept is to find a stable spatial configuration of groups of agents. The mutual attitudes of the agent pairs are known and represented in the form of the so-called Moreno matrix. This matrix may be defined by an expert or researcher in a flexible manner by means of linguistic expressions that describe the degree of acceptance or antipathy. The agents can constantly move across the plane starting from a randomly generated layout. This movement is *driven by success.* The rules controlling the movement and each step payoff are also defined by natural language-like phrases formulated according to an expert knowledge. Such expressions are called linguistic patterns (LPs). The virtual social forces generated by these patterns exert an influence on agents and affect their behavior.

In specific application problems, one has to create LPs that reflect logical relationships and describe the desired state of the examined system in reality. Since a formal description by classic mathematical formulae may be difficult due to the information uncertainty, the fuzzy sets and LPs appear to be well fitted to this job. The determination of the appropriate patterns can be obtained, for instance, by finding a consensus between knowledge of different experts within the given field or in concrete situations.

This paper was inspired by works on modeling migration behavior [1–3] and the initial concept of linguistic patterns (LPs) proposed in [4,5]. Although the presented idea shares some assumptions and properties similar to those of previous works, there are several profound differences. The novelty and specific distinctive features of our proposal that constitute a contribution of this research are highlighted below.

- To the best of our knowledge, the LPs and linguistic variables, which are at the core of our study, have not been previously used in this research area. This novel solution, which allows in a flexible way to encode and apply expert knowledge, made it possible to resign from rigid mathematical relations that describe the agents' behavior.
- In other studies, regular grids are usually used to model migration behavior. Their particular cells specify possible agents' locations, which is a substantial restriction. In our approach, it is possible to place agents at any point in an

* Correspondence to: Faculty of Management, Wrocław University of Science and Technology, 27 Wybrzeże Wyspiańskiego st., 50-370 Wrocław, Poland.
E-mail addresses: jerzy.grobelny@pwr.edu.pl (J. Grobelny), rafal.michalski@pwr.edu.pl (R. Michalski).

https://doi.org/10.1016/j.knosys.2022.108497
0950-7051/

unconstrained plane. It is feasible thanks to exploiting an analogy to physical attractive and repulsive forces.
– The resulting flexibility and the freedom of agents' movement allow for a significant extension of the proposed approach. Our method goes far beyond the analysis of agent behavior in the classical sense. It is, for instance, possible to obtain the so-called scattered plots which is unfeasible in classical migration models due to the mentioned limitations. Our approach enables the construction and analysis of such scattered plots for a variety of criteria and tasks. Thus, it can be used much more widely used in economic and social practice than traditional techniques. In this paper, the proposed idea is presented along with simple exemplary models that show some of its possible applications and highlight its potential flexibility.
– In the context of searching the space of possible solutions to find the best one, the assessment criteria defined in the form of LPs provide an additional advantage over other methods. According to the axioms of fuzzy set theory and multivalued logic, the proposed criterion of the mean truth of LP fulfillment cannot exceed the value of one for a specific solution. This feature is not available in classic approaches. Therefore, when the maximum value is obtained, one can be sure that there exists no better solution. There can be, however, some other equivalently good configurations, given specific LPs expressions.

The remainder of the paper is organized as follows. First, we review the relevant literature that outline the background of our proposal and highlight differences with other studies. Next, in Section 3, we present our approach and explain its relations with fuzzy set theory and concepts of LPs. Their application to define our virtual forces and evaluation criteria are illustrated with party participant configurations inspired by [6]. The following section demonstrates the application of our methodology to classic migration simulations introduced by Sakoda [1] and Schelling [2,3]. The final stable configurations generated by the proposed LP-based method take the form of *scattered plots*. Therefore, in Section 6, we show and compare them with solutions provided by suboptimal eigenvector-based *scattered plots*, generated according to the classic Drezner [7] idea. Then, we further discuss the simulation results obtained in illustrative examples and provide a broader scope of potential applications of our proposal's potential applications. In addition, we outline possible future extensions and research directions. The article ends with concise conclusions.

2. Background and literature review

2.1. Agent-based modeling

Agent-based modeling (ABM) is still very popular, with new approaches constantly being explored and developed in various fields. For example, such techniques are currently being used to model the dynamics of online social networks. An extensive discussion on the use of agent-based methods in this area is presented by Lymperopoulos and Ioannou [8]. Some of the recent advances in modeling social phenomena are discussed in [9]. Castro et al. [10] reviewed more than 60 proposals of using agent-based models to determine climate mitigation policies. Agents are also employed in systems-of-systems analysis. Recent developments in this area are described by Silva and Braga [11].

Other exemplary reviews on agent-based modeling concern product lifecycle [12], collective intelligence [13], education [14], decision making in manufacturing [15], sociotechnical energy transitions [16], and even mosquito behavior and disease [17]. The ongoing pandemic led some researchers to also focus on this type of modeling technique, for instance, [18], and a review in this field [19]. North [20] gives some theoretical aspects of agent-based models along with the analysis of recent free software environments that support this approach.

2.2. Moving agent-based modeling with crisp knowledge representation

A special case of ABM is moving agent-based modeling (MABM). Since it constitutes the direct background for our proposal, we provide a brief review of the key advances and the most important research trends in this area.

The macrolevel analysis of social group behavior based on interactions and local displacements of agents *living* on a checkboard was suggested by Sakoda [1]. In his concept, agents inhabit cells of an 8×8 checkboard. The mutual attitude for each pair of agents can be defined as neutral (0), positive (+1), or negative (−1). Because not all cells are occupied, in subsequent steps, each agent can move to a neighboring free place where it would feel better than at the location he left. The movements of the individual agent in a given step are restricted to a 3×3 square with the current agent position at the center. The *feeling* in a particular place is defined by *valence*, which is computed as the sum of attitude indicators between the moving agent and all other agents divided by the sum of distances between them. The main objective of Sakoda's analysis was to identify group formation processes by analyzing various parameters of the model, particularly segregation and suspicion. The Sakoda concept represents the trend in modeling the dynamics of social system dynamics based on cellular automata (CA) theory. The proposed checkboard model and the rules of agent interactions in the local environment (Moore's neighborhood in CA theory) clearly refer to the idea of CA. The history, potential, and advantages of implementation of this methodology in the dynamics of analysis of the social systems dynamics have been discussed, for instance, by Hegselmann and Flache [21].

Schelling [2,3] tackled group behavior modeling in a similar way, with a strong focus on segregation processes. Just as in Sakoda's study, the simulation of the group formation process, with agents deployed on a regular grid, was based on the local interactions and movements of agents within Moore's neighborhood (3×3). In basic experiments, the agents were divided into two classes. The objective of the agents' movements was to increase the proportion of individuals in the agent's own class to foreign individuals in the new *habitable* location (Moore's neighborhood cell). The Sakoda and Schelling modeling concepts were developed prior to CA computer implementations, so this study involves only small populations and follows simple rules.

The development of computer modeling has boosted interest in this research field. It resulted in a number of models and computer programs that allowed free modification and analysis of their parameters. Hegselmann and Flache [21] presented various examples of the applicability of the Sakoda and Schelling analysis. They also involved the possible implementation of different parameter definitions to model social group formation processes.

In the spirit of CA, Klüver and Stoica [22] presented a variety of social group behavior simulations based on the definitions of agents' attitudes, specified by sociomatrices, also called Moreno matrices [23]. The arrays allow for taking into account attitude dynamics of different intensities, also those asymmetric. This was a significant extension in comparison to earlier approaches where only binary classification was possible. Klüver and Stoica [22] demonstrated the correlation between the dynamics of the agents' behavior on the CA grid and the matrix structure with the Moore neighborhood size. They also proved a far-reaching compatibility of this approach with the models based on neural networks and genetic algorithms.

Hegselmann and Flache [21] proposed the application of CA as a tool for modeling and understanding social dynamics. They used CA to illustrate the potential for extended implementation of Schelling and Sakoda models. Beltran et al. [6] proposed a model that allows the analysis of the behavior of a small group of agents during a party held in a closed room. The movements of agents are modeled by changing the cells on the grid, similarly to CA models. The agent's decision to move at a given time is determined by the level of dissatisfaction with the agent's current position. Agents move to minimize the level of dissatisfaction calculated as a function of the discrepancy between the actual and desired distance between them. This indicator takes into account concepts of personal and social zone distances derived from sociology (Hall [24]).

The analytical perspective on group behavior dynamics based on local interactions also prevails in the observation of pedestrian traffic on the streets. This approach is applied in modeling evacuation procedures from public buildings. Differential games are the main method used in such an analysis of agent mobility. Hoogendoorn and Bovy [25] proposed a model of pedestrian flow in which the movement of agents is determined by the respective equations. They are based on physical parameters modeled as a multimolecular system and empirical pedestrian behavior. In this way, they determine the particular decisions that influence the acceleration and direction of movement. Contrary to CA models, here agents are constantly on the move, and their movement is limited only by infrastructure parameters like the sidewalk width, pedestrian crossings, etc. The application of a similar approach to the analysis of crowd behavior during evacuation is presented by Helbing et al. [26]. In such circumstances, the movement of the crowd is primarily determined by the herd behavior. The interactions between agents in the model are mainly *physical*, so the movement is described by differential equations based on precise physical parameters. The authors demonstrated the applicability of that model to the analysis of facilities safety design, such as the width of emergency cross-passes, the shape and size of corridors, etc. Gao et al. [27] proposed a more recent approach in this field that uses the classic agent-based simulation system to evaluate urban management strategies. Most MABMs use grids, but recently one can encounter more often less rigorous approaches. For example, evacuation models [28] and pedestrian studies in urban spaces [29] replicate actual physical traffic routes along with relevant constraints and surrounding objects.

The models described so far demonstrate trends based on analysis of the agent movement dynamics in a real, geographical, or physical space. Similar analysis may also be applied in an arbitrary or abstract universe. For instance, Moya et al. [30] tried to understand the influence of terrorist attacks on elections in Spain. In the proposal of Bin and Zhang [31], the movement of agents takes place in the universe of *degrees of loyalty to the group*. In this model, the simulation of group behavior consists in observation of agents' attitudes towards the company's policy on social or economic incentives. The migration of agents in an abstract space is analyzed by CA also in the works of Yu and Helbing [32] and Helbing et al. [33]. The authors demonstrated a significant impact of agent migration opportunities on the formation of separate groups of cooperators and defectors (in the *prisoner's dilemma* game) and the self-organization of cooperative clusters at the macrolevel. Another approach in this trend is proposed by Kowalska-Styczeń & Sznajd-Weron [34]. The authors showed how the movements of agents in the space that simulate the locations of various sources of information affect the efficiency of person-to-person communication.

2.3. Soft knowledge representation in agent-based modeling

The models discussed in the previous subsections are based on knowledge represented in strict mathematical formulas and physical relations that involve crisp calculations. As our method uses soft knowledge representations, recent developments in this area are covered in this subsection.

There exists a relatively small, but systematically increasing, number of studies that include the representation of imprecise or rough knowledge in general ABM. In an early study, Ma & Nakamori [35] employed a simple fuzzy linear quantification method to represent and aggregate data on the properties of objects for Kansei Engineering purposes. Their ABM involved both objective and subjective information. The proposal to merge ABM with fuzzy logic was also presented by Iandoli et al. [36] in the context of individual and collective learning processes. They represented agents' opinions as fuzzy variables and combined them using the ordered weighted averaging operator [37].

Interesting concepts of incorporation of fuzzy relations to ABM in social simulations such as matchmaking were discussed by Hassan et al. [38]. Their ideas were further extended to model friendship dynamics in [39] and lately in [40]. Martínez-Miranda and Pavón [41] proposed using ABM simulations of human behavior to create effective and efficient work teams. They applied soft knowledge to determine and represent the agent emotional state and trust with respect to its team members. Other models regarding the workforce that include fuzzy approaches were also developed in a different context by Raoufi et al. [42], Kedir et al. [43], or most recently in [44].

Some of the latest developments that blend soft knowledge with AMB take advantage of fuzzy cognitive maps. This notable trend is presented, for example, in the works of Mei et al. [45], Giabbanelli et al. [46], or Mehryar et al. [47]. Recently, one can also find AMB approaches focused on decision making that employ 2-tuple fuzzy variables [48,49], or the fuzzy logic controller (FLC) [50].

While the literature using soft-knowledge-based modeling in ABM is growing in size and importance, its application to MABM is scarce. Among the few works in this area, there are those of Sharma et al. [28] as well as Yıld ız & Çağdaş [29]. Both works present models of moving agents in real physical spaces, that is, closed rooms and urban spaces, respectively. In these proposals, the crisp rules and formulae are used in conjunction with the soft knowledge. The authors applied the same core idea of the FLC concept [51] to take advantage of the soft knowledge in some components of their models. In general, the idea of FLC consists in creating a system of rules that control the behavior of objects over time. These rules describe the relations between fuzzy variables that represent the input and output of the mapped system. Such relations are developed on the basis of expert knowledge and/or observations of the system behavior. The rules take the form of a set of specific if-then phrases. The knowledge represented in this way allows one to infer the shape of the response to a given set of input parameters.

Sharma et al. [28] used the FLC framework to generate the agent's rate of movement towards the exit in the room evacuation process. This parameter is generated in the model based on the panic level, stress, physical weight of the agent, and its distance from the exit. The final agent movement process is modeled by a complex mechanism that combines a genetic algorithm with a neural network. The resulting movement direction also takes into account physical constraints in the room.

In turn, Yıld ız & Çağdaş [29] proposed a MABM to simulate pedestrian traffic in urban space. In this approach, the FLC concept was used to determine the attractiveness of architectural objects in the environment. The level of attractiveness generated

Table 1
Agents' mutual attitudes.

Agent number	1	2	3
1	×	Positive Big (PB)	Negative Big (NB)
2	×	×	Positive Medium (PM)

the force of attraction that acted on moving pedestrian agents. The rules for controlling this force included fuzzy levels of variables such as distance, heating, population, and illuminance. The movement of each agent was also subject to forces resulting from their current geometric position relative to other agents and possible obstacles. These other forces are calculated in the model according to simple, strict geometric and physical rules represented as crisp mathematical formulas.

The described above works take advantage of some soft knowledge representation in the form of FLC that is one of many components of MABM, however, they do not involve any LPs. Two features that seem to particularly distinguish these works from the classical approaches are the free geometric space of movement and the flexibility in formulating rules and relationships that define agent movement. Our proposal fits generally into the trend, as it combines properties of traditional moving agent models and less formal expert knowledge given in the form of natural language-like expressions called LPs. In contrast to using soft knowledge to only selected relations that affect agent movement ([28,29]), LPs in our framework control the behavior of agents completely by generating all virtual forces. Unlike in the classical FLC approach, where agent's movement parameters are defined at each step, LPs are a description of the desired state of the whole system. The virtual forces acting on the agent in our framework are directed to decrease the distance of the system from this ideal state at each step.

The concept of LPs was initially introduced by Grobelny [4,5]. In principle, an LP is a logical expression to which a particular degree of truth can be ascribed. The LP can act as a quality criterion in an appropriately defined system. Grobelny [52], Raoot & Rakshit [53], and Grobelny & Michalski [54] applied LPs to object layout optimization. In the Grobelny [55] study, LPs were used to obtain the hierarchy of objects based on fuzzy pair wise comparisons. The possibility of using expressions similar to natural language in the form of LPs enables their application to optimization problems with imprecise knowledge. The use of LPs with fuzzy set relations along with the relaxation of certain moving constraints makes our proposal unique and potentially interesting for further scientific and practical research. To the best of our knowledge, there are no proposals that include both linguistic variables, multivalued logic, and LPs in this area. In the next section, we describe our concept in detail and provide the mathematical formulas necessary to understand its implementation.

3. The concept of linguistic patterns and virtual force

The approach proposed in this paper involves the implementation of LPs in the modeling of success-driven agent movements. The following example illustrates the essential components of the LP application to MABM. It is inspired by the idea put forward by Beltran and Salas [1] regarding simulations of party attendees' behavior. In this scenario, three agents initially found themselves in a rectangular room where the party takes place. The configuration is presented in Fig. 1.

After the official ceremony, the agents begin to move from their initial random positions. Their movements depend on known relations that reflect their mutual attitudes. They may be expressed in a way shown in Table 1.

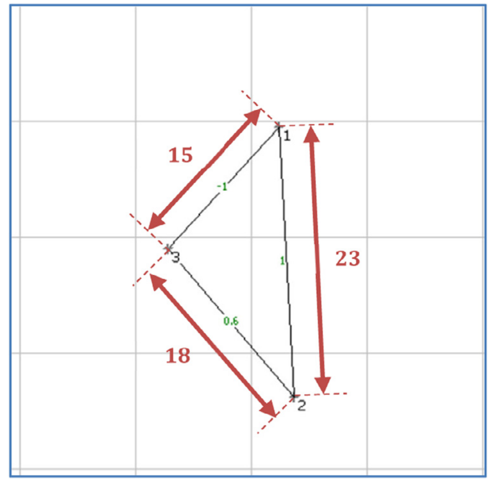


Fig. 1. Initial position: 3 agents with distances presented as a percentage of the maximum distance, i.e., the length of the longer side of the room. The dimensions of a single cell in a grid amount to 10% of the maximum distance.

The expressions presented in Table 1 are linguistic exemplifications of the symmetric Moreno matrix (Klüver and Stoica [22], Moreno [23]). They simply mean that agents 1 and 2 like each other very much, agents 1 and 3 dislike each other very much, and agents 2 and 3 like each other moderately. In circumstances such as the said party, the pairs liking each other would tend to be close to one another, while the resentful ones would prefer to stay away. Therefore, we may use the following statements to describe the configuration presented in Fig. 1 and its future dynamics. *If I like the person, the distance between us is small* and *If I dislike the person, the distance between us is large*. Such statements may be expressed in a slightly more formal manner as patterns presented in (1) and (2):

$$\text{P1: IF Attitude}(i, j) \text{ is POSITIVE, THEN } j \text{ is at a SMALL\_DISTANCE from } i \quad (1)$$

$$\text{P2: IF Attitude}(i, j) \text{ is NEGATIVE, THEN } j \text{ is at a LARGE\_DISTANCE from } i \quad (2)$$

Both patterns define the attitude-desired distance relationship for each pair of agents analyzed. The difference between them is related to the nature of their mutual impact. Pattern (1) corresponds to attraction, whereas pattern (2) relates to repulsion between agents.

These formulas are examples of LPs that, in the context presented, constitute the agents' *wellbeing* criteria. The levels of such *wellbeing* depend on the respective criteria values. They need to be determined for each agent in a given configuration. The values can be specified as the degree of truth for a particular pattern. Practical calculations can be based on the formula for determining the truth of the implication proposed by Łukasiewicz (Grobelny [55]). If $t(l)$ and $t(r)$ denote the degree of truth on the left and right side of the implication, respectively, then the truth value is computed by formula (3).

$$\text{Truth(P1)} = \min[(1 - t(l) + t(r)), 1], \text{ only for non-negative attitudes between agents } i, j. \quad (3)$$

The same formula applies to Truth(P2), which is calculated only for negative attitudes between agents i, j. It may be noticed that (3) is a kind of generalization of the truth value table related to a classic implication. There are more such generalizations possible, e.g., those discussed by Dubois and Prade [56]. Certainly, the

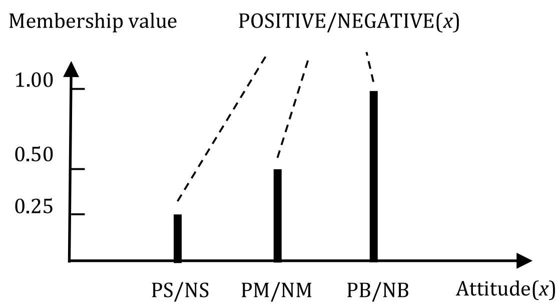


Fig. 2. Exemplary definitions of truth values for the patterns (1) and (2) expressions (POSITIVE or NEGATIVE) for three levels of linguistic expressions (PS — POSITIVE_SMALL, NB — NEGATIVE_BIG, etc.).

determination of the truth value according to formula (3) requires t(l) and t(r) to be specified. The theory of fuzzy sets provides simple and intuitive tools for that purpose (Zadeh [57,58]). For example, t(l) for patterns (1) and (2) denotes the truth value in the linguistic expressions 'Attitude(i, j) is POSITIVE' and 'Attitude(i, j) is NEGATIVE'. We can treat the linguistic expressions presented in Table 1 as fuzzy sets (singletons) with appropriate values of the membership function. Then, these values of the membership function may be regarded as degrees on truth of the left side of patterns (1) or (2). An illustration of this approach is given in Fig. 2. The linguistic expressions in Table 1 are arranged only on an ordinal scale and the exact distances between successive items are unknown. However, the assumption of a proportional increase of the truth value for successive phrases listed on this scale seems reasonable. It is especially true and justifiable when additional information on how such expressions are perceived by humans is not available.

The concept of a possibility proposed by Zadeh [28,29] is a generalization of a simple, direct assignment of the truth value for linguistic expressions. This measure allows for the determination of truth of the LP fulfillment using an expression represented by a fuzzy set (e.g., a fuzzy number) in a given universe. The aforementioned criterion may be formally expressed as follows.

$$\mathrm{Truth}(l) = \mathrm{POSS}(\mathrm{Attitude}(a, b) \text{ is POSITIVE} | \mathrm{Attitude}(a, b) \text{ is A}). \tag{4}$$

If one defines the expression Attitude(a, b) as a linguistic variable represented by fuzzy sets in the universe of discourse $X = (x_1, \ldots, x_n)$, POSITIVE(x) and A(x) in (4) represent this realization of the variable as fuzzy sets in this universe, then the truth is computed according to formula (5):

$$\mathrm{Truth}(l) = \max_x(\min_x(\mathrm{POSITIVE}(x), \mathrm{A}(x))). \tag{5}$$

Formula (5) represents the consistency of two expressions POSITIVE and A, which are the pattern and a specific realization for a given pair, respectively. In other words, (5) exhibits the possibility of the fact that A is POSITIVE. By analogy, we can define the truth value for the NEGATIVE(x) case. Fig. 3 presents exemplary definitions of LPs from Table 1 expressions. We also illustrate how the calculation of the possibility measure Truth(l) works for fuzzy representations of linguistic values. In Zadeh's original proposal [57,58], this measure determines the *degree of truth* of the fulfillment of a given criterion by specifying its level of magnitude. The criterion can also be expressed as a linguistic expression.

Fig. 3 shows a generalization of the approach presented in Fig. 2 where the degree of truth is directly determined. The particular significance of this extension lies in enabling experts to specify fuzzy sets that represent linguistic expressions with their concrete spaces of variability, characteristic for a given context. If we assume that the linguistic values from Table 1 are defined in the form of fuzzy sets in the numerical space of 0–10 ratings (e.g., based on interviews), then Fig. 3 shows how to determine the value of Truth(l) for a given PM(x). Here, we assume a linear membership function for POSITIVE(x).

Although Fig. 3 defines LPs using fuzzy sets in an artificial numerical universe, the same could be performed using more objective information. For example, universe X may represent the number of interpersonal contacts in a given period, or the proportion of common views, etc. Then, Truth(l) is simply the value of the POSITIVE(x) function for a given x value.

The determination of the truth value for the right side of the patterns t(r) requires that appropriate functions be defined for the following statements: '(j) is at a SMALL_DISTANCE from i' and 'j is at a LARGE_DISTANCE from i'. Examples of such definitions as fuzzy sets are shown in Fig. 4.

In this case, LPs are represented as fuzzy sets defined in the universe of distances determined as percentages of the maximum distance. Although the shape of the function is intuitive, it can reflect objective knowledge of the human perception of distance in a given context. For example, such an objectified knowledge may refer to the notions of personal and social spheres (Hall [24], Beltran and Salas [6]).

The definitions described above allow for the determination of truth values for patterns (1) and (2). Therefore, the appropriate calculations can be performed for each agent from the party presented in Fig. 1. Let us consider the situation of agent 1. To assess his *well-being*, we should use pattern (1) to specify the relation with agent 2 because the attitude is positive, and pattern (2) to obtain the relation with agent 3 as the attitude is negative. Based on Table 1, Fig. 2, Fig. 4, and using pattern P1 (1) to assess the relationship with agent 2, we arrive at the following results:

t(l) = 1, because the attitude value is PB (POSITIVE_BIG) and the degree of truth for the left side of pattern (1) (POSITIVE Attitude) is 1.

t(r) = 0, because the truth value for the right side of pattern (2) (SMALL_DISTANCE) for the distance between agents 1 and 2 (greater than 0.2) is 0.

Then,:

Truth_P1(1, 2) = min ((1 – 1 + 0), 1) = 0.

Similarly, using pattern (2) for the assessment of the relation with agent 3, the results are as follows:

t(l) = 1, because the attitude value is NB and the degree of truth for NEGATIVE is 1.

t(r) = 0.75, because in Fig. 4, the distance of 0.15 between agents 1 and 3 is DISTANCE_LARGE to the degree of 0.75.

Therefore,

Truth_P2(1, 3) = min ((1 – 1 + 0.75), 1) = 0.75.

The total level of satisfaction of agent 1 with the configuration shown in Fig. 1 can be assessed by calculating the average of the truth values for both patterns. In this example, it is 0.38. Changing the location of agents may improve this evaluation. Moving agents in directions that cause an increase in the truth values of the patterns appears to be reasonable to achieve this goal. It can be assumed that agent 1 senses a kind of unique *virtual* attraction or repulsion to agents 2 and 3 at a given moment. Feelings are proportional to the truth levels of patterns (1) and (2). Such an approach is a simplified analogy of the concept of *social force* described by Helbing [26]. The vector of the force ($\overrightarrow{\mathrm{VF}}$) acting on agent 1 lies on a straight line that connects agents 1 and 2. It has a length of $\mathrm{VF}(1, 2)^+ = 1 - \mathrm{Truth\_P1}(1, 2) = 1$ and is directed towards agent 2, which denotes its attractive nature. Let us analyze the force of magnitude $\mathrm{VF}(1, 3)^- = 1 - \mathrm{Truth\_P2}(1, 3)$. In the position presented in Fig. 1, the vector length is 0.25 and

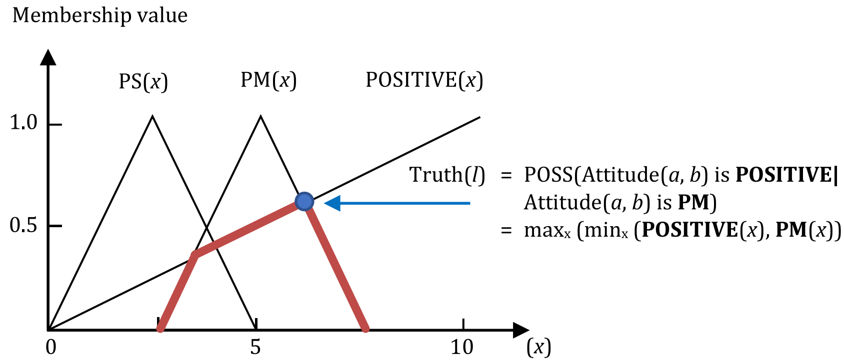


Fig. 3. Graphical illustration of possible definitions of linguistic variables in an artificial numerical universe of discourse and calculation of Truth(l) according to Eq. (5).

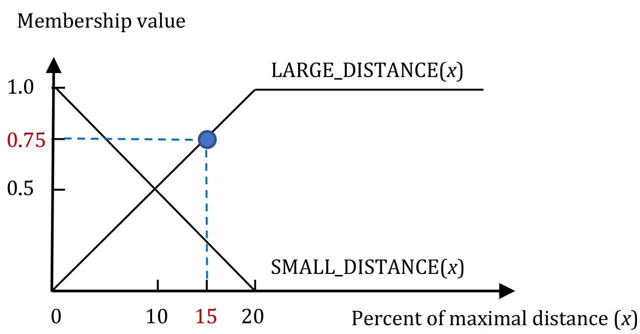


Fig. 4. The pattern expressions LARGE_DISTANCE and SMALL_DISTANCE are interpreted as fuzzy sets in the universe of the maximal distance percentages.

lies on the straight line connecting agents 1 and 3. The direction of this force is opposite to the position of agent 3. This means that the force has a repulsive character.

Assuming that the agent's response to the force being felt is motion, the displacement distance in one step is proportional to the magnitude of this force. Since the maximum force value between a pair of agents is 1, the appropriate parameter s may define the physical distance of the displacement. It can be expressed as a percentage of the longer side of the rectangle. It may also be reasonable to associate the value of s with a physical limitation of displacement, e.g., possible speed. Taking into account both forces and assuming $s = 0.1$, the agent moves in the direction that is the sum of both vectors, and ultimately reaches a new position denoted as 1'($\leftarrow 2^+$, $\leftarrow 3^-$) in Fig. 5.

The precise determination of successive positions of the agents consists of performing simple calculations based on geometrical dependencies. For example, to obtain the geometric position of agent 1 after acting with an attracting force between agents 1 and 2, i.e. VF$(1, 2)^+$ one needs to do the following.

(a) Calculate the distance (DIST) between agents 1 and 2 for their current coordinates (x_1, y_1) and (x_2, y_2) according to (6):

$$\mathrm{DIST}_{(1,\ 2)} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \tag{6}$$

(b) Calculate new coordinates for agent 1 $\left(x'_{1(\leftarrow 2^+)},\ y'_{1(\leftarrow 2^+)}\right)$.

Since $cos\,(\alpha) = \frac{x_2 - x_1}{\mathrm{DIST}_{(1,\ 2)}} = \frac{x'_{1(\leftarrow 2^+)} - x_1}{s\cdot \mathrm{VF}(1,\ 2)^+}$, and $sin\,(\alpha) = \frac{y_2 - y_1}{\mathrm{DIST}_{(1,\ 2)}} = \frac{y'_{1(\leftarrow 2^+)} - y_1}{s\cdot \mathrm{VF}(1,\ 2)^+}$, where α is the angle between the positive horizontal axis (x) and vector $\overrightarrow{\mathrm{VF}\,(1, 2)}^+$, we arrive at the direct formulas (7):

$$\begin{aligned} x'_{1(\leftarrow 2^+)} &= x_1 + s \cdot \mathrm{VF}(1,\ 2)^+ \cdot \frac{x_2 - x_1}{\mathrm{DIST}_{(1,\ 2)}}, \\ y'_{1(\leftarrow 2^+)} &= y_1 + s \cdot \mathrm{VF}(1,\ 2)^+ \cdot \frac{y_2 - y_1}{\mathrm{DIST}_{(1,\ 2)}}. \end{aligned} \tag{7}$$

These computations must be performed for all links of each agent. The final position in a given step is the vector sum of these partial displacements. In the same way, the displacement characteristics of each agent are computed with respect to the appropriate criterion represented by an LP.

The presented idea of the individual agent behavior can be applied to all agents. Thus, each of them, at a given moment, can see others and perform the same calculations. As a result, according to the rules presented, appropriate displacements take place in consecutive steps. It eventually generates dynamic movements of agents. Such an approach derived from the party example may reflect the behavior of small social groups.

It can also be observed that the distance covered by an agent in a single step in the simulation depends on the number and strength of relations with other agents. Then, taking into account the forces coming from all agents, the displacement of the agent being analyzed in a single step should not exceed the total values. Therefore, in the proposed algorithm (Appendix A), a quotient of s by n is used, where n is the number of agents.

Once this procedure is completed for each agent in a given step, the mean truth value for the patterns of all agents is determined by formula (8). It can be interpreted as a measure of satisfaction for the entire group of agents in a given configuration.

$$\mathrm{Mean\_truth} = \frac{\sum_{i=1,\ j=i+1}^{m} Truth\_P1(i, j) + \sum_{k=1,\ l=k+1}^{u} Truth\_P2(k, l)}{p} \tag{8}$$

In the above formula (8), m is the number of agent pairs with non-negative attitudes, u – the number of pairs with negative attitudes, p is the number of all agent pairs, and $m + u = p$. In general, $p = \frac{n^2 - n}{2}$, where n is the number of agents. Since for pairs of unrelated agents $t(l) = 0$, the truth values for patterns P1 and P2 equal 1. In situations where there are a considerable number of unrelated pairs of agents, those ones would artificially increase the mean truth. Therefore, when calculating the mean truth, it is reasonable to take into account only the pairs of agents for which $t(l) > 0$. In this way, our indicator will only refer to agents remaining in any relation with others. In this case, the value p denotes the number of only linked agent pairs.

The movement of agents is finished when each VF(i, j) for each agent amounts to a zero vector or after performing a specified number of steps. This concept can be described in the form of a simple algorithm. The pseudocode is provided in Appendix A.

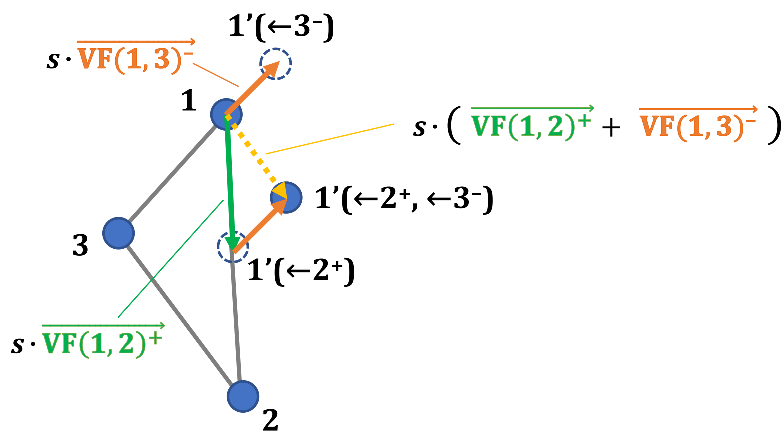


Fig. 5. Vector representation of virtual forces and the agent 1 movement step in the configuration from Fig. 1. Notation 1'(←2^+) depicts the position of agent 1, if only the attractive force from agent 2 is taken into account; 1'(←3^-) denotes the location of agent 1 determined only by the repulsive force from agent 3; 1'(←2^+, ←3^-) is the final position of agent 1 resulting from forces generated by both agents 2 and 3.

In addition to the principles presented above, the possibility of defining the radius r to determine the *range of vision* for all agents was also introduced. Each agent examines the relations only with those agents who remain within a distance shorter than r, which is expressed as a percentage of the maximum distance. In our model implementation in Delphi, agents are graphically represented by numbered crosses or squares of equal sizes. The analyzed area is defined as a rectangle of any size and in any unit. The distances within such an area are expressed as percentages of the longer side of the area. The step coefficient s that represents the distance of the agent's dislocation in a single step of the simulation in response to VF of length 1, is also defined as a percentage of that maximal distance.

Given defined patterns and variables, the simulation of the agents' movements always starts with a random distribution on the plane. After the random agents' locations are generated, the distances between each pair are calculated, allowing the computation of the truth values. The application tracks the movement of selected agents from their initial positions to end positions, as shown in Fig. 6a, and records the Mean_truth values for all agents in each step (Fig. 6c).

The figures presented demonstrate that, in the discussed case, the system reaches the complete truth for the defined patterns and their parameters in approximately 45 steps. However, it is easy to conclude a priori that such a simple implementation of the proposed concept shall not be considered universal. It can be noted that if there are only positive values in the attitude matrix (Table 1), then the optimal solution would result in more or less dense clusters. Such a behavior is observed regardless of the level of these positive values, and the final configuration depends on the definition of the distance adopted in P1. An alternative optimum result would involve all agents located at one point. As it is considered to be rather unlikely behavior, the proposed framework and its application were supplemented with the possibility of introducing additional, reasonable patterns and virtual forces they generate. The concepts and basic properties of our model are further discussed on the basis of several illustrative examples.

4. Impact of model parameters on agent behavior

The presented framework provides flexibility in formulating both input data and agent behavior rules. The influence of key parameters on the formation of the resultant configurations of agents can be conveniently analyzed using examples. It is convenient to select problems in which reasonable or optimal stable layouts can be predicted in terms of accepted criteria or patterns. For this purpose, two test matrices with specific structures were created to represent the relations between agents.

It was assumed that for each pair of agents, the degree of mutual positive relationship reflected the number of daily personal contacts such as emails, phone calls, conversations, etc. The number of interactions ranged from 0 to 10. It was also assumed that the truth degree of t(l) for P1 is defined as shown in Fig. 7. It is linearly dependent on the number of contacts (Fig. 7a), or the truth of any number of contacts larger than 0 is taken as 1. It should also be noted that in this structural context, the relations and their degree of truth need not be strictly dependent on mutual personal attitudes but rather on the more substantive cooperation of the agents.

The two exemplary structures are shown in Fig. 8. Degrees of truth t(l), presented in green, were calculated according to the function in Fig. 7a for randomly assigned contact numbers. When the function in Fig. 7b is used, the truth value will be equal to 1. Analogously to the party example, one can analyze the spatial behavior of agents initially placed at random locations, in particular, their final stable configurations. By maximizing the truth value of pattern (1), we should obtain structures in which pairs of agents with stronger relationships are closer to each other. One can also notice that similar criterion is also used to search for optimal solutions in facility layout problems (Grobelny & Michalski [54]).

The dynamics of agent movement and the resulting layout configuration for the P1 pattern will be shaped by the perception of SMALL_DISTANCE. Fig. 9 shows a simple formulation of the truth t(r) for the SMALL_DISTANCE expression in the universe of *Percent of maximal distance*.

This definition makes it possible to reset the strength of attraction for each pair of agents if the distance between them is less than the *afuzzy* value. Consequently, using this value, the model can adopt the concepts of social and personal proximity proposed by Hall [24]. They were initially implemented in the context of MABM by Beltran et al. [6]. These concepts take into account the dissatisfaction of people resulting from strangers crossing a certain distance in interpersonal relationships. They can be understood as a small protective sphere that an organism tries to keep between itself and others (Hall [24]). Depending on the context, culture etc., the radius of this comfort zone is estimated to be 1.5–4 ft (personal distance) and 4–12 ft (social distance).

A series of simulations of agent behavior were run with the SMALL_DISTANCE definition from Fig. 9 and setting $afuzzy = 0.05$,

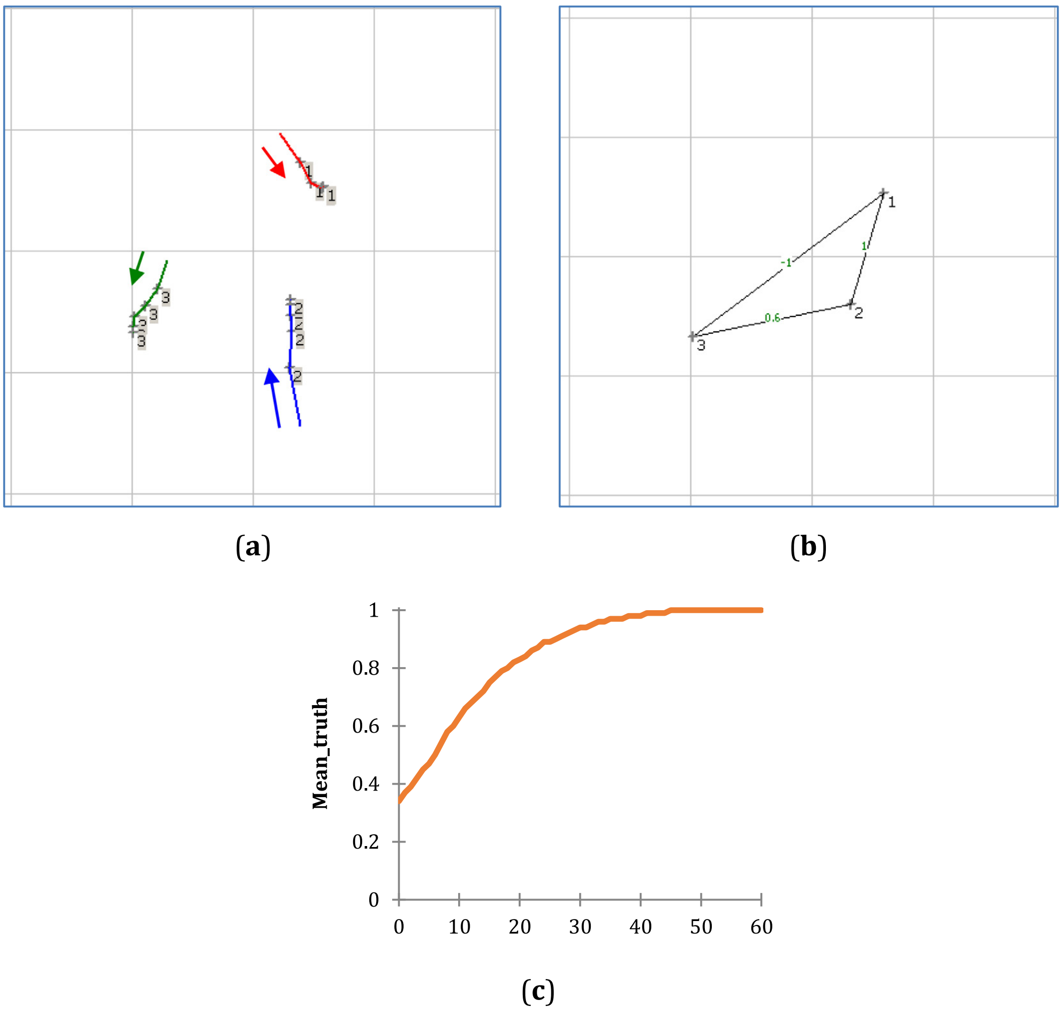


Fig. 6. Agents' movement patterns from the configuration shown in Fig. 1. The definitions of the truth value for the patterns correspond to Figs. 2 and 4. The grid size is 0.1 (10% of the maximal length). Parameter $s = 0.05$ (5% of a maximal distance) (a) Movements towards the stable configuration. Locations of agents marked by every 10^{th} step; (b) The final stable arrangement; (c) Dynamics of the Mean_truth value in subsequent steps.

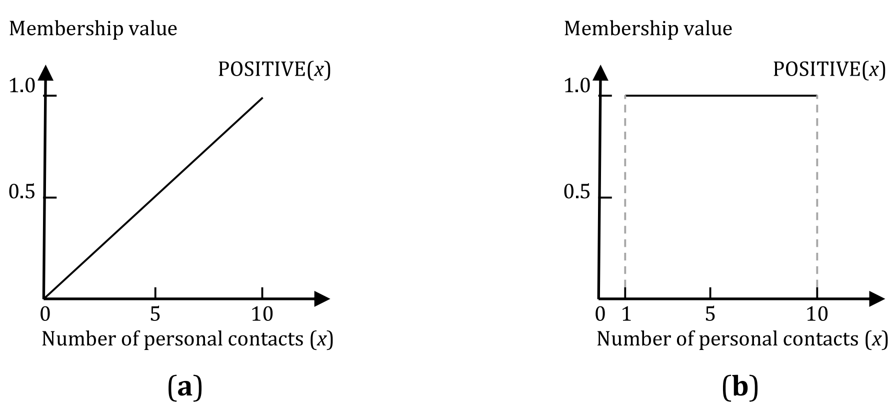
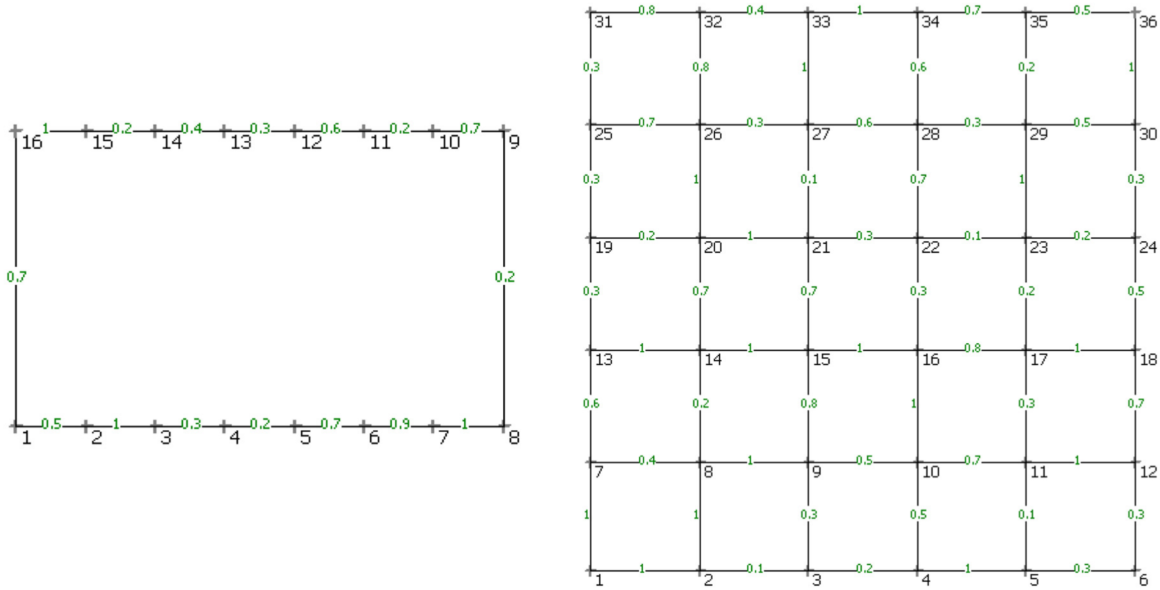


Fig. 7. Two definitions of the degree of truth in relations between agents.

bfuzzy = 0.1. The relationships between agents were taken from the structures in Fig. 8a and 8b. We applied a linear function to assess the value of t(l) (Fig. 7a), and set the value of s between 0.01 and 0.1. The highest values for Mean_truth in the resulting configurations were obtained for $s = 0.05$, yielding layouts similar to those shown in Fig. 10. All agents were clustered in a small area around the center of the plane such that the distances between them were essentially proportional to the t(l) values that represent their relations (Fig. 8a).

Note that in the resulting configuration in Fig. 10a, the anticipated and desirable proximity of the agents' locations is maintained (Fig. 8a). However, there are also some close accidental neighborhoods, for example, between unrelated agents 3 and 9 or 5 and 1. For the relationships in Fig. 8b, the resulting layouts



(a) **(b)**

Fig. 8. Two exemplary structures of t(l) relations with easily predictive solutions. (a) Circular arrangement; (b) Grid arrangement.

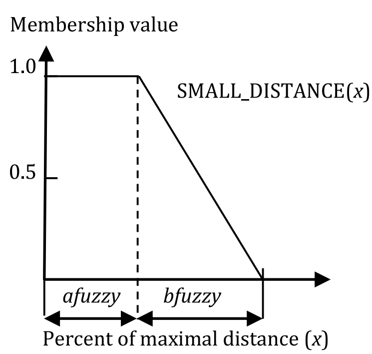


Fig. 9. Schematic definition of a SMALL_DISTANCE expression in the universe of *Percent of maximal distance*.

(Fig. 10b) are even denser. This is because the length of the personal/social distance represented in pattern P1 by *afuzzy* only affects the distance of related agents. Since agents defined as mutually *neutral* do not interact with each other during their movements, it seems reasonable to propose an additional pattern that incorporates the notion of social or personal distance for them. We propose to construct this LP in the following way (9):

P3: IF Attitude(i,j) is NEUTRAL THEN Distance(i, j)

is PERSONAL/SOCIAL_DISTANCE (9)

The scheme to determine the truth value of PERSONAL/SOCIAL_DISTANCE is defined in Fig. 11. As in the example shown above, the membership function reflects the degree of truth of achieving the *personal distance* with respect to the distance between a pair of agents. Likewise in pattern P2, it generates a corresponding *repulsive* virtual force, proportional to the deficit of truth in pattern P3 (9).

In the developed implementation, *afuzzy* and *pd/sd* can be set independently. The *afuzzy* parameter is equivalent to personal distance, but it is only applicable to agent pairs that are in a relationship according to the P1 or P2 patterns. The P1 pattern generates an attractive force (P2 – a repulsive force) only until a pair of agents reaches a distance with the value of *afuzzy*. In contrast, *pd/sd* creates pattern P3 that is applicable only to pairs that are not connected by any relation. P3 generates a repulsive force until the unrelated agent pair reaches a distance of *pd/sd*.

Another simulation experiment conducted included the third pattern (9). In this modified approach, *pd/sd* was set to be three times greater than *afuzzy*. It reflects the assumption that unrelated agents should rather remain at a social distance that is about three times the personal distance. The value of *pd/sd* set here was inspired by the concepts of personal and social distances presented in the work of Hall [24]. The authors claim that the social distance of approximately three times greater than the personal one is statistically desirable. It applies to people who are not in a relationship with each other.

The resulting layouts were qualitatively different from previous simulations. Fig. 12 demonstrates these configurations of agents that achieved *complete satisfaction* with Mean_truth = 1. The simulation involved pattern P3 with *pd/sd* = 0.15. It started with the same random agent layouts as in previous experiments shown in Fig. 10. All other parameters were unchanged. The final Mean_truth value for pattern (P3) is recorded and can be included in the overall Mean_truth evaluation. Figs. 12c and 12d illustrate the dynamics of Mean_truth values for patterns P1 and P3.

The Mean_truth value for P3 can be treated as a separate measure or quality criterion of a given configuration. It can also be appropriately combined with Mean_truth for P1 and/or P2. The definition of truth shown in Fig. 7b allows us to evaluate attitudes or relations in binary terms (1–0, true–false). The described experiments (*afuzzy* = 0.05, *bfuzzy* = 0.1, *pd/sd* = 0.15, s = 0.1) demonstrate the surprising reasonable dynamics of the presented approach. The final layouts obtained for the tested relationship structures (Fig. 8 with t(l) = 1 for each link) are shown in Figs. 13a and 13b.

In both examples, the resulting layouts preserve the desired structure of agent proximity and form very regular patterns. The dynamics of changes in truth values for patterns P1 and P3 are shown in Fig. 13c and 13d. Both approaches similarly achieve a stable configuration within approximately 25 simulation steps, reaching full truth for pattern P1 and a truth value of 0.98 for pattern P3.

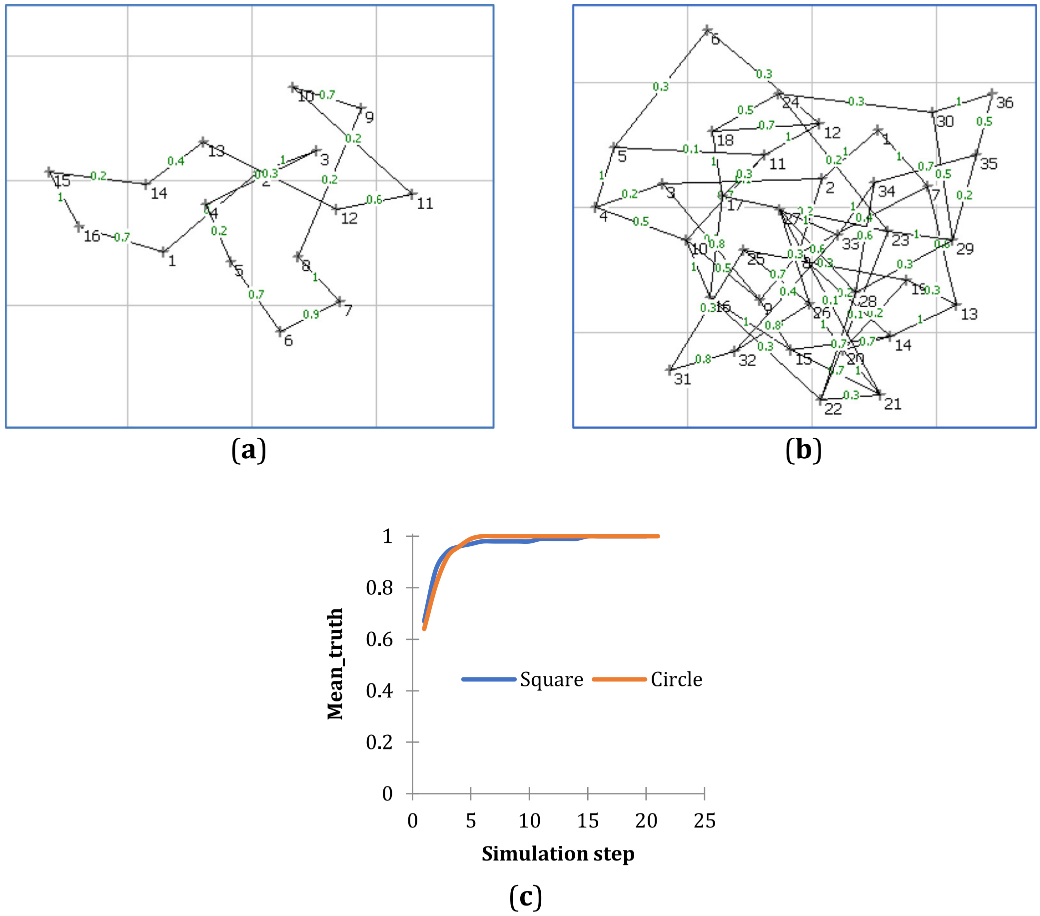


Fig. 10. Agent configuration in experiment 1 with t(l) values for the relations ($afuzzy$ = 0.05, $bfuzzy$ = 0.1, s = 0.05). Mean_truth = 1 (left and right). (a) Circular link structure; (b) Grid link structure; (c) Mean_truth value dynamics.

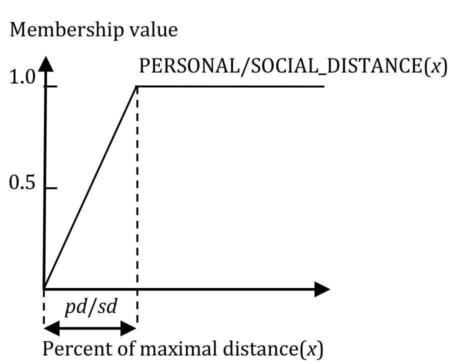


Fig. 11. Schematic definition of PERSONAL/SOCIAL_DISTANCE (pd/sd).

5. Segregation examples

The proposed method of modeling the movement of agents based on *a party* example may seem specific and of limited practical use. However, its flexibility, resulting from the freedom to define patterns and their parameters, gives many more possibilities. For example, it seems that the proposed method can be applied to model the dynamics of many social processes determined by mutual attitudes of agents. For this purpose, Sakoda [1] proposed a social interaction checkboard model, which allows us to analyze the movement of agents on a regular grid. In the Sakoda model, members of two groups live on a checkerboard. They have positive, neutral, or negative attitudes towards each other, called valences. These values are defined by integers. Individuals have the opportunity to move to empty cells in their 3×3 neighborhood. If there are no empty cells, an individual can jump over one cell. Migration is always local or is allowed only within certain limits. Individual i uses the migration option to move to locations where (10) is maximized.

$$\sum_{i=1,\ j=1(j\epsilon P)}^{n} \frac{V_{ij}}{\left(d_{ij}^{2}\right)^{\frac{1}{w}}} \tag{10}$$

In formula (10), V_{ij} denotes the valence of individual j for individual i, P is the set of all individuals when not all cells are occupied, d is the Euclidean distance between i and j, and w determines how strongly the valences are discounted by distance. In this model, all agents can *see* each other. Sakoda's world is an 8×8 checkerboard occupied by two groups, each with 6 members. Members of one group are represented as squares, and members of the other group are represented as crosses. Sakoda analyzes different combinations of attitudes. He calls one of them segregation, another suspicion. These attitudes are shown in Table 2.

Schelling [2,3] proposed a different approach to segregation. The concept of his model differs from Sakoda's approach in that agents act only on the basis of local observations that involve at most eight nearest neighbors. The number of neighbors to whom the agent has a positive attitude determines the satisfaction with a given location. The agent may decide to move to the nearest free cell on the grid if the number of *positive* neighbors at the new location is greater than this number at its current location. The relations for this segregation model are included in the last column of Table 2.

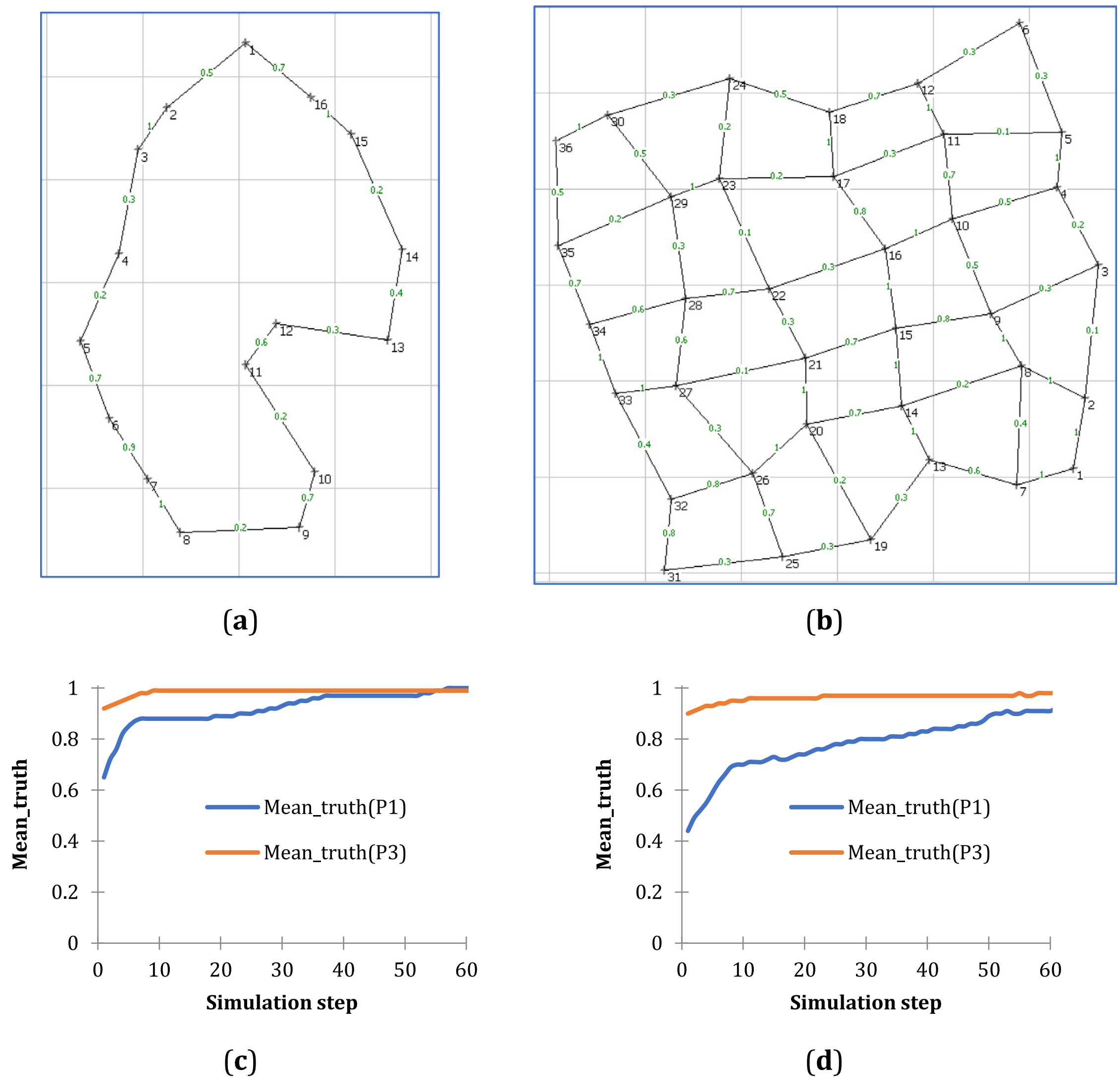


Fig. 12. Final layouts for agents with parameters from Fig. 10 with added pattern P3. (a) Circular link structure; (b) Grid link structure; (c) (d) Mean truth value dynamics for the patterns P1 and P3 for the circular and grid link structure, respectively.

Table 2
Attitude combinations for suspicion and segregation models.

Attitude	Sakoda suspicion		Sakoda segregation		Schelling segregation	
	Squares	Crosses	Squares	Crosses	Squares	Crosses
Squares	0	−1	1	−1	1	0
Crosses	−1	0	−1	1	0	1

The solutions of the original research of Sakoda and Shelling are schematically shown in Fig. 14. Analysis of the above models from the perspective of LPs and virtual forces required the determination of appropriate model parameters. In the proposed implementation, it is necessary to determine the appropriate agentvisibility, that is, the distance within the range of which patterns and generated forces are exercised. Such an approach is a type of local environment estimation in regular grid-based models. In this approach, each agent can analyze patterns and is subjected to forces only inside a circle with a radius equal to the visibility range. Of course, the corresponding truth values for patterns are calculated only with respect to this range. Patterns P1 and P2 are, respectively, involved instead of formula (10).

To examine the behavior of agents whose *attitudes* are defined similarly to the Sakoda and Schelling models, a value +1 or –1 in Table 2 was assumed to indicate compliance with patterns P1 and P2, respectively, to the degree of 1 (t(l) = 1).

Fig. 15 shows the corresponding fuzzy definitions of distances used in the analyzed patterns. There were 10 experiments conducted for the matrix corresponding to the suspicion and segregation models. The experiments involved 36 agents. The purpose of this study was to test the behavior of agents with attitudes corresponding to Sakoda models with four different visibility ranges (r = 0.1, 0.3, 0.5, and 1). For each r, the displacement process started with the same random layout of agents. The resulting configurations with the highest Mean_truth(P2) values are shown in the screenshot excerpts displayed in Fig. 16.

A direct comparison of the results obtained in this experiment with the model results in Fig. 14 is not possible, but some qualitative observations can be made. In general, agents clustered into groups with individuals neutral towards each other. Assuming that agent mobility is based on global observations (r = 1), the resulting model will predict the formation of clearly isolated groups, similar to the original solutions in a regular grid. These groups are somewhat more concentrated in the case of segregation, which is the result of attraction forces generated by the LP used in this case. Fig. 17 presents the simulation results for Schelling's model (Table 2, column 3). Simulations were performed for the same values of r as for the Sakoda models. Since the configuration in all runs for r = 0.5 is the same as for r = 1, it is not presented in the figure.

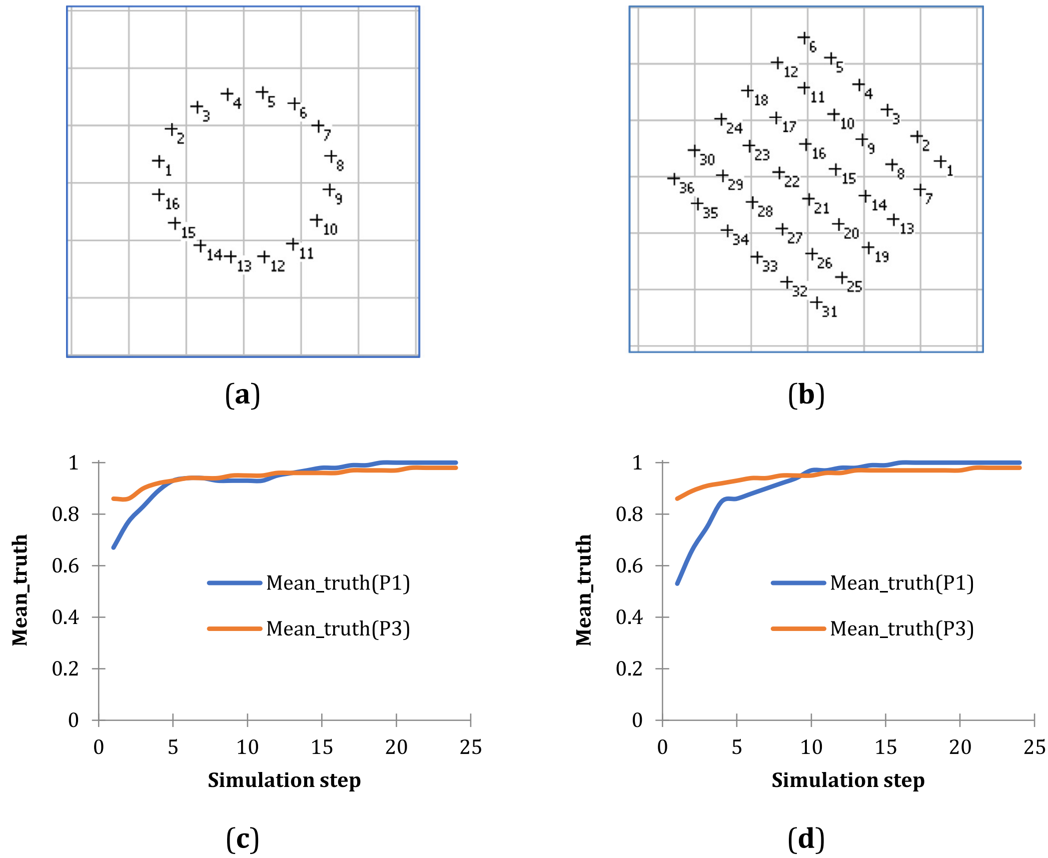


Fig. 13. Best final layouts of two cases analyzed in Fig. 8 with all truths of links $t(l) = 1$. (a) Circular link structure; (b) Grid link structure; (c) (d) Mean truth value dynamics for patterns P1 and P3 for circular and grid link structure, respectively.

Although, as in the previous example, a direct and precise comparison of the results of our approach with those obtained by Schelling on a regular grid (Fig. 14) is not possible, a qualitative analysis is feasible. Assuming that the mobility of agents is based on local observations ($r = 0.1$, 0.2, and 0.3), the resulting configuration predicts the formation of increasingly isolated groups. Similarly as in the original, regular grid model. In each case, the model reaches a stable configuration in 40–70 steps. Fig. 18 demonstrates the dynamics of the mean truth value in the types of relations studied, under the assumption that all agents can see each other (r = 1).

In the experiments presented above, the agents moved according to the established patterns with fixed parameters. The fundamental element that determines the achievement of a stable configuration is the assumed perception of the distance defined in each pattern. To qualitatively illustrate the importance of the assumed perceived distance parameters, the effect of a radical change in the parameter *pd*/*sd* on the stable configuration shape is shown in Fig. 19. As mentioned above, the parameter *pd*/*sd* defines an acceptable distance between individuals in different social situations. In this sense, it can be implemented directly in the party scenario described above.

The analyzed segregation models are related to social groups migrations, where a direct interpretation of *pd*/*sd* in its literal sense is not applicable. However, it can generally be assumed that the relation between the *afuzzy* and *pd*/*sd* parameters describes to some extent the unified *us-them* attitudes in the analyzed group of virtual agents. Using Schelling's model of relations (Table 2, column 3), simulations were performed for 72 agents. They started from the same random layout and situation where agents do not tolerate the proximity of strangers.

The *pd*/*sd* value was assumed to be 4 times greater than *afuzzy*. In the second experiment, the opposite was assumed: the *pd*/*sd* value was twice lower than *afuzzy*. Such a relation may reflect the actual curiosity or openness of some social groups. Fig. 19 shows the resulting stable virtual community configurations, assuming that agents can see 20% of the environment with ($r = 0.2$) and 100% ($r = 1$), respectively.

Although it might not be noticed at first glance in the figure presented, the structures resulting from different values of *pd*/*sd* are qualitatively entirely different. This effect of the *pd*/*sd* parameter on the results consists in a clear variation in the homogeneity of the groups formed. In parts (a) and (b) of Fig. 19, where high values of *pd*/*sd* were applied, the groups are highly homogeneous. They consist exclusively of squares or crosses. In parts (c) and (d) of Fig. 19, which depict results obtained with considerably lower value of the *pd*/*sd* parameters, mixed groups are formed. An individual group includes both squares and crosses.

The dynamics of the mean truth value (Fig. 19e) for P1 in the case where $r = 1$, suggests that a stable configuration is obtained in approximately 30 steps and is similar to that presented in Fig. 18 for 36 agents.

6. Comparison with scattered plots

The examples analyzed so far lead to stable agent configurations that maximized the average truth value for specified patterns with certain parameters. Since they take the form of *scattered plots*, it seems interesting to compare the properties of these configurations with the results of Drezner's classical approach to the construction of suboptimal scattered plots for facility layouts [7]. As the author writes, they may be applied in various areas: '*They proved to be very useful, for example, in the*

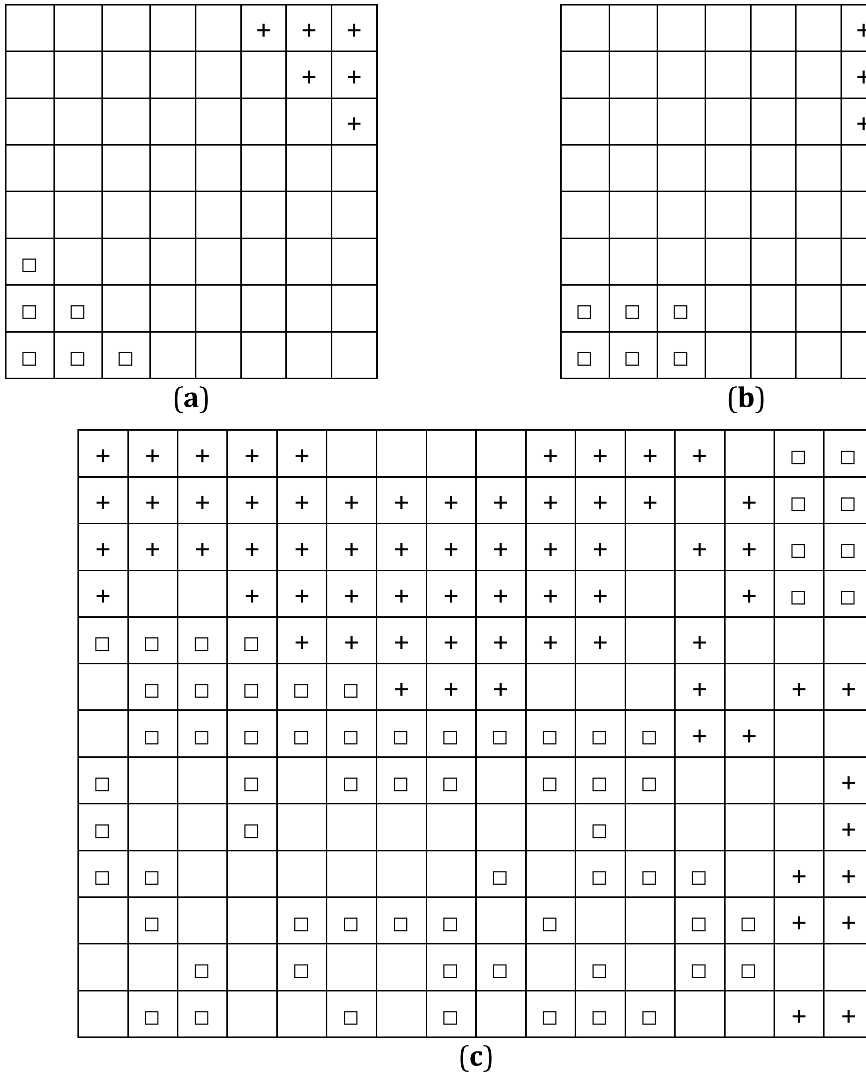


Fig. 14. Final configurations of Sakoda and Schelling models. (a) Sakoda suspicion; (b) Sakoda segregation; (c) Schelling segregation.

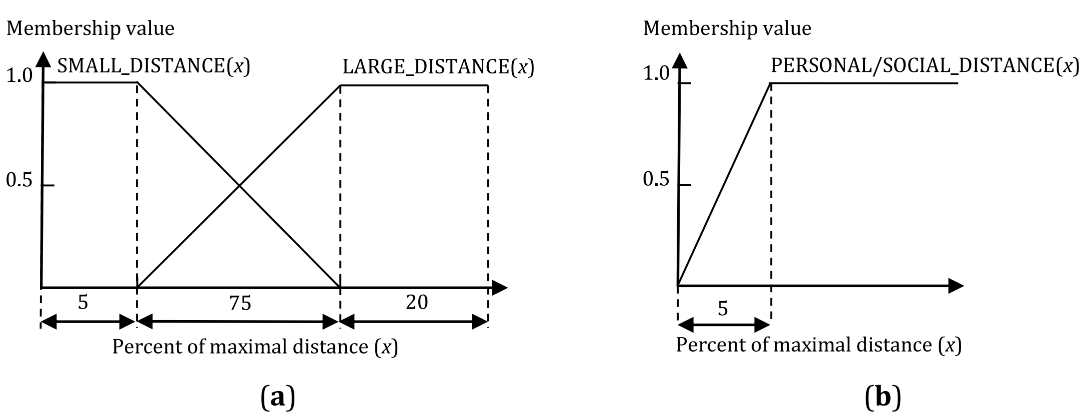


Fig. 15. Definitions of distances used in segregation experiments.

work of architects. Drawings of facilities scattered on a plane may be a useful benchmark for them in urban planning, locating industrial plants, etc.'.

The starting point for the construction of scattered plots proposed in [7] is the definition of the objective function to minimize, specified according to (11).

$$f = \frac{\sum_{i=1,\ j=1,\ \ i\langle\rangle j}^{n} c_{ij} \cdot d_{ij}}{\sum_{i=1,j=1}^{n} d_{ij}}, \tag{11}$$

In formula (11), c_{ij} denotes the link (intensity of interaction) between facilities i and j, while d_{ij} denotes the distance between them. The proposed heuristic is very effective and is based on the properties of eigenvectors and the eigenvalues of matrices.

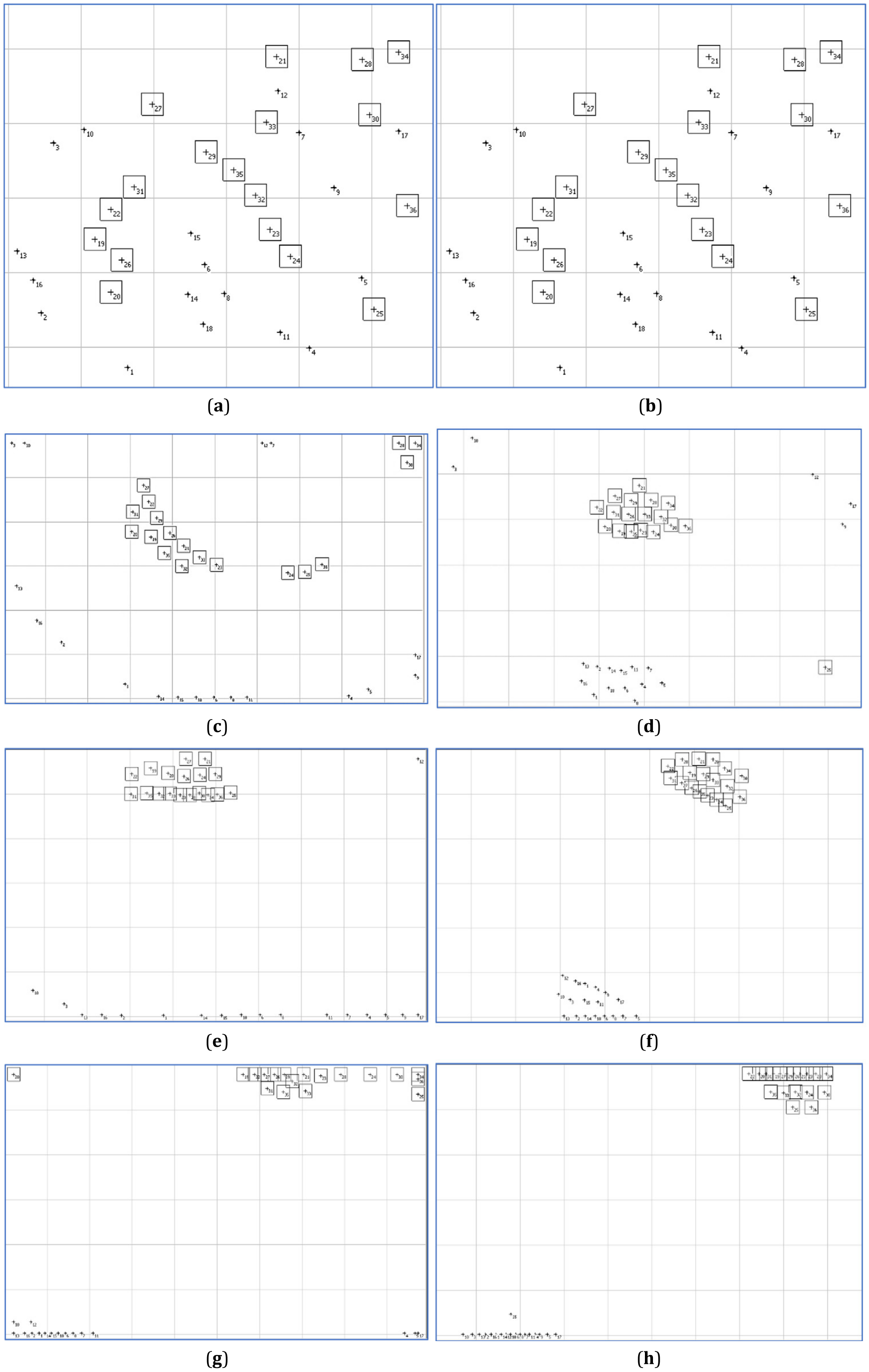


Fig. 16. Suspicion (left column: (a), (c), (e), (g)) and segregation (right: (b), (d), (f), (h)) experiment results for different neighborhood ranges r. (a) $r = 0.1$, Mean_truth(P2) = 0.98, Mean_truth(P3) = 1; (b) $r = 0.1$, Mean_truth(P2) = 0.99; Mean_truth(P3) = 1; (c) $r = 0.3$, Mean_truth(P2) = 0.99; Mean_truth(P3) = 0.99; (d) $r = 0.3$, Mean_truth(P2) = 0.99; Mean_truth(P3) = 0.96; (e) $r = 0.5$, Mean_truth(P2) = 0.99; Mean_truth(P3) = 0.97; (f) $r = 0.5$, Mean_truth(P2) = 0.99; Mean_truth(P3) = 0.93; (g) $r = 1$, Mean_truth(P2) = 0.91; Mean_truth(P3) = 0.94; (h) $r = 1$, Mean_truth(P2) = 0.98; Mean_truth(P3) = 0.86.

Namely, if in formula (11) d_{ij} is replaced by d_{ij}^2, which Drezner considers '*intuitively reasonable*', we have formula (12).

$$ff = \frac{\sum_{i=1,\ j=1,\ \ i\langle\rangle j}^{n} c_{ij} \cdot d_{ij}^2}{\sum_{i=1,j=1}^{n} d_{ij}^2}. \tag{12}$$

Eq. (12) has its optimal solution in a straight line. The x coordinates of the solution are successive elements of the eigenvector associated with the second smallest eigenvalue of matrix **S** where $s_{ij} = -c_{ij}$ for $i = j$ and $s_{ii} = \sum_{j=1}^{n} c_{ij}$ for all i. The y coordinates are elements of the eigenvector associated with the third smallest

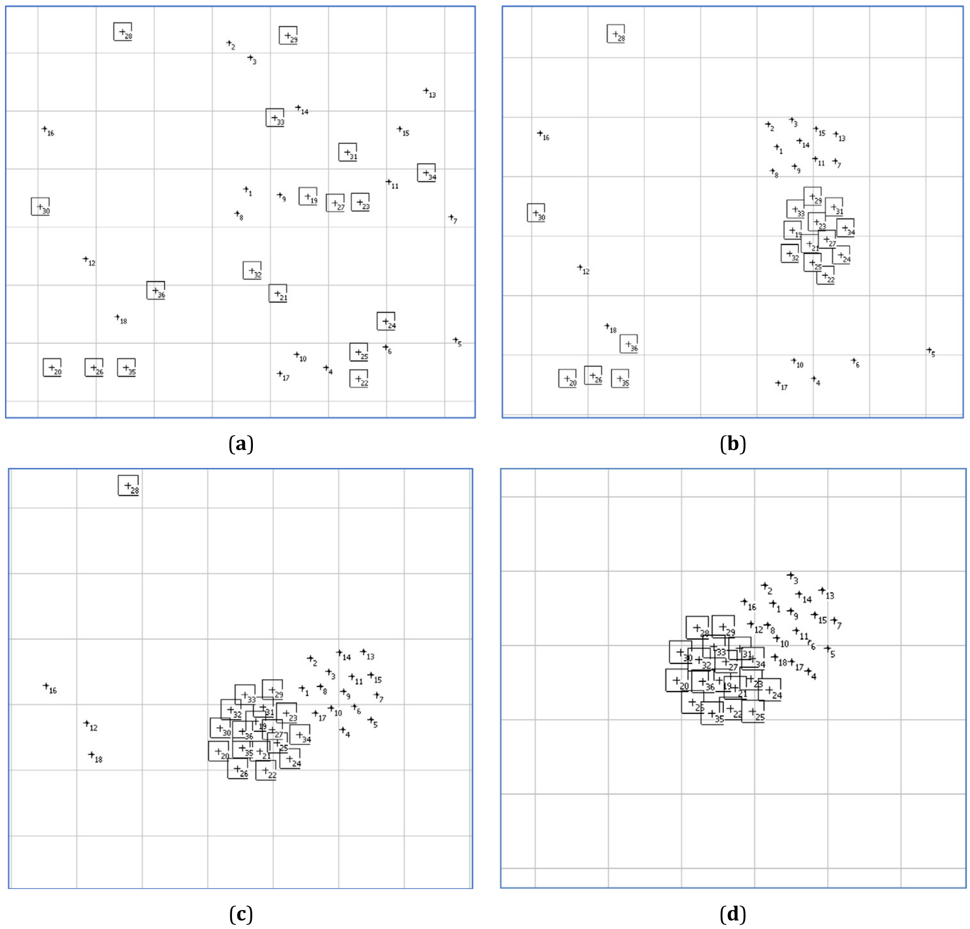


Fig. 17. Stable configurations of the Schelling model for different values of r. (a) $r = 0.1$, Mean_truth(P1) $= 0.98$, Mean_truth(P3) $= 1$; (b) $r = 0.2$, Mean_truth(P1) $= 0.97$; Mean_truth(P3) $= 1$; (c) $r = 0.3$, Mean_truth(P1) $= 0.97$; Mean_truth(P3) $= 1$; (d) $r = 0.5$ and $r = 1$, Mean_truth(P1) $= 0.97$; Mean_truth(P3) $= 1$.

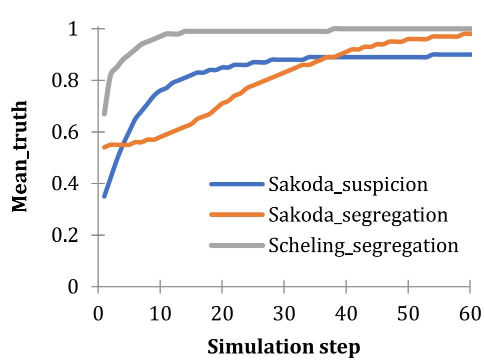


Fig. 18. Mean truth values dynamics (P1 or P2) for the examples studied.

eigenvalue of matrix **S**. Thus, the algorithm for obtaining a suboptimal solution is relatively simple. Having the set of links c_{ij}, it is enough to: (i) construct the matrix **S**; (ii) calculate all eigenvalues and associated eigenvectors for **S**; (iii) select two eigenvectors associated with the second and third lowest eigenvalues, treat them as the coordinates x and y of the solution on a plane, and calculate objective function values.

The implementation of the algorithm described above allows for a comparative analysis of examples taken from Drezner's work that demonstrate the potential of the LP-based approach in relation to the eigenvector approach. Table 3 presents three cases of searching for optimal scattered plots in terms of the objective function (12). In the matrix of relations between agents, pairs that should be close to each other are denoted by 1, and by zeros otherwise.

In Fig. 20, the first row shows the suboptimal scattered plots obtained using the Drezner approach. Agent configurations for the defined examples were based on agent relations expressed as the truth value t(l) using the proposed Drezner method. The respective patterns (1) and (3) were applied, assuming the distance definition consistent with Fig. 9 where $afuzzy = 0.05$, $bfuzzy = 0.1$, and $pd/sd = 0.05$ is taken from the graph in Fig. 11. The best configurations resulting from 10 trials are presented in the second row in Fig. 20.

In these configurations, the mean truth values for the applied patterns and the value of f were calculated according to Drezner's method. Analyzing the results from the perspective of the f function, the plots obtained using the LP approach (row 2) are worse than those obtained using eigenvectors. However, the configurations shown in the second row are similar to those in the first row in terms of the neighborhood structure of the agents. They are characterized by high average truth values of the P1 and P3 patterns. In addition, they show a very uniform distribution of agents.

This is undoubtedly due to the similar perception of mutual personal/ social distance in this model. In practical applications for objects layout purposes, these distances can reflect, for example, the size of objects or the required dimensions of space allocated for agents' activity. The eigenvector approach does not take into account the sizes of objects in question.

The last row in Fig. 20 provides interesting results obtained in the LP approach. To obtain scatter plots for this row, a

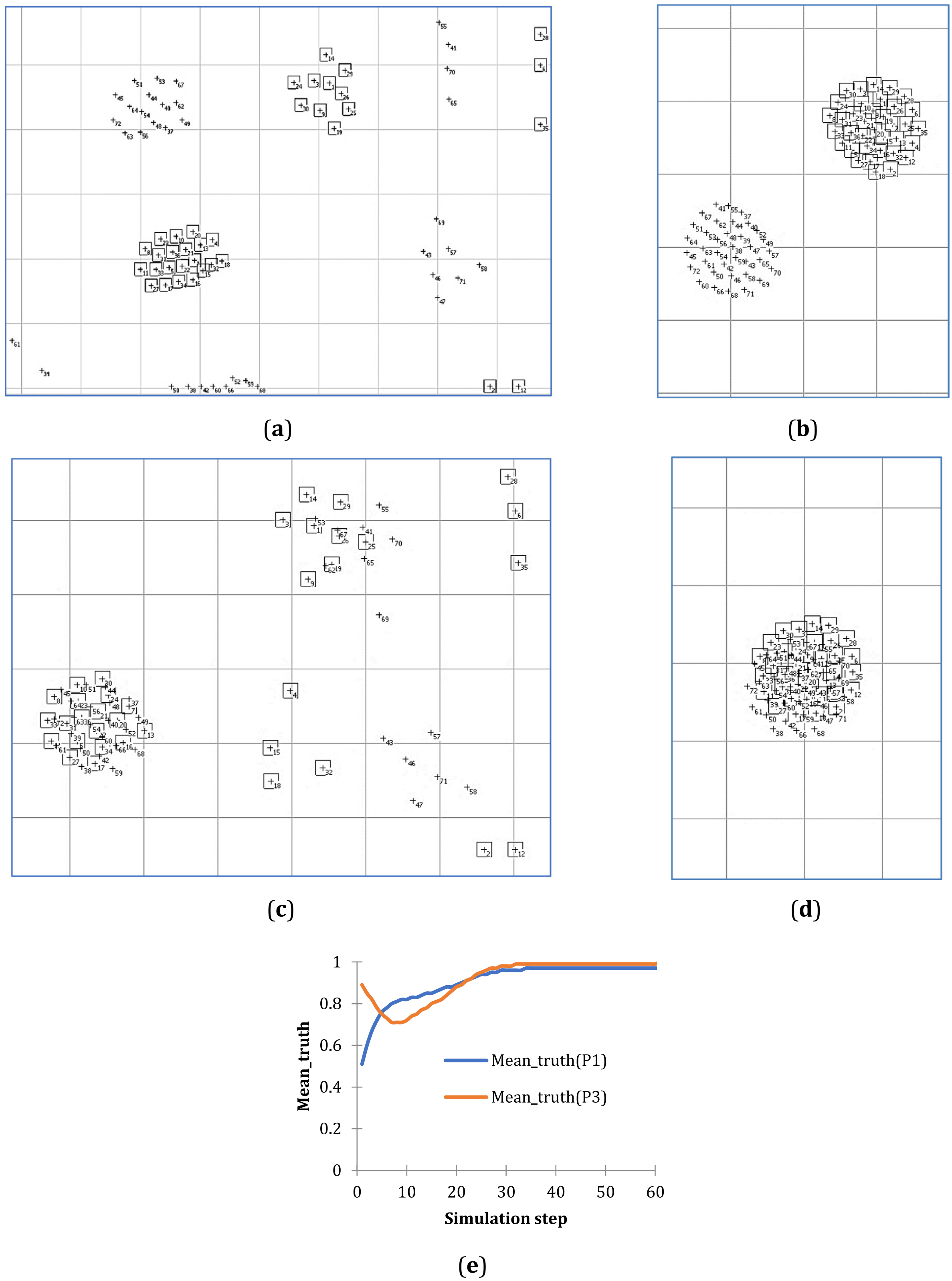


Fig. 19. Schelling's model simulation results for *afuzzy* = 0.05, *bfuzzy* = 0.5 for two different levels of *pd*/*sd* {0.2, 0.025} and *r* {0.2, 1}. (a) *r* = 0.2, Mean_truth(P1) = 0.96, Mean_truth(P3) = 1; (b) *r* = 1, Mean_truth(P1) = 0.97; Mean_truth(P3) = 1; (c) *r* = 0.2, Mean_truth(P1) = 0.96; Mean_truth(P3) = 1; (d) *r* = 1, Mean_truth(P1) = 0.96; Mean_truth(P3) = 1. (e) Mean_truth dynamics of P1 and P3 for configurations presented in (a), (b), (c), (d) with *r* = 1 and *pd*/*sd* = 0.2.

series of pilot experiments were conducted. They aimed to establish such parameters for patterns P1 and P3 that were expected to act similarly to Eq. (11) in computing the objective function.

It is worth noting that the value of f decreases as the denominator increases. Thus, in principle, the best configuration of agents is the one in which related objects are close together and unrelated objects remain distant. In successive trials, the distance

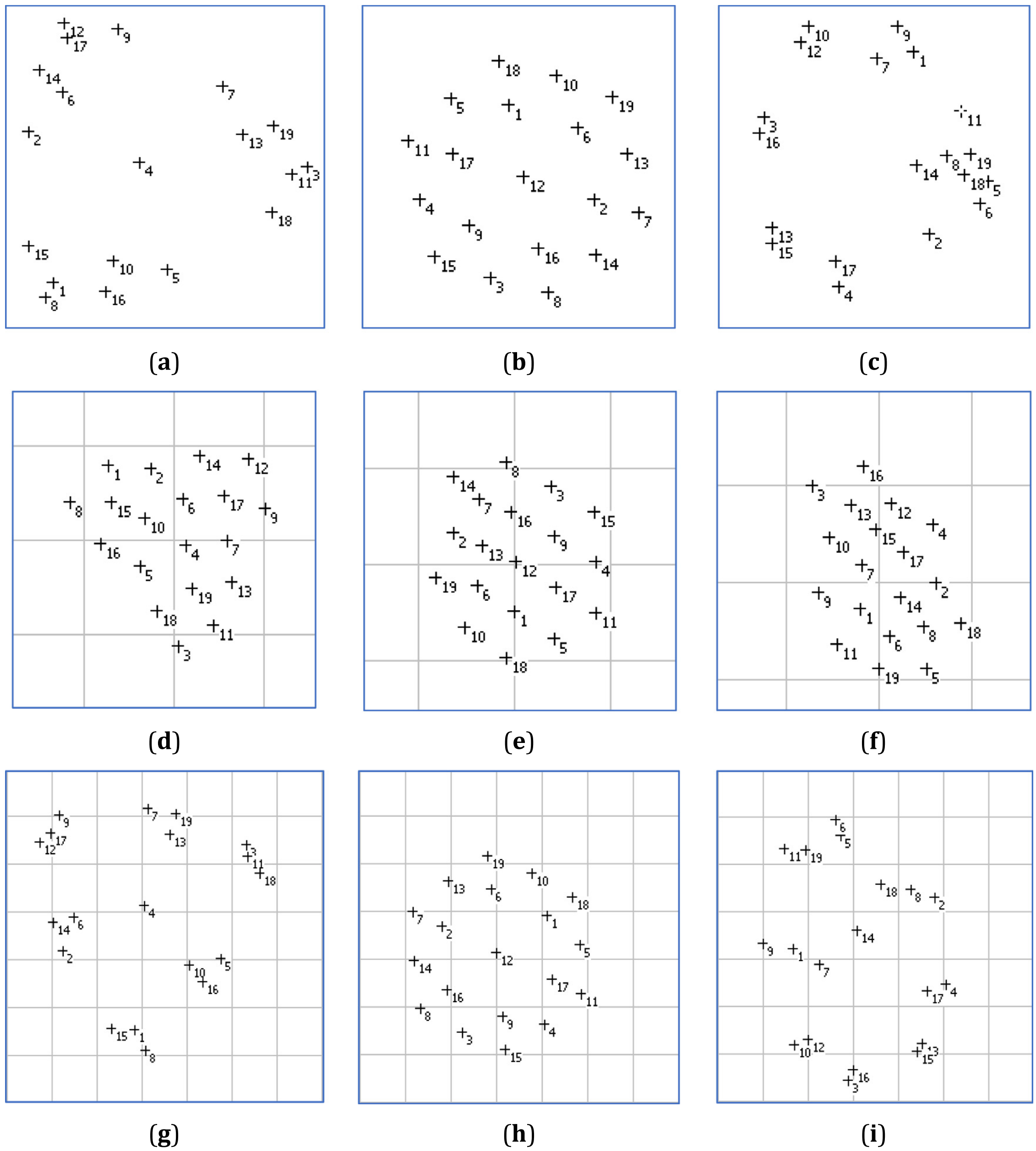


Fig. 20. Configurations obtained for Drezner examples with different parameters. The first row, that is, (a), (b), (c) presents scattered plots obtained by Drezner's approach. The distances in these plots are defined relatively and the physical scale is irrelevant. Hence there is no reference to the experimental area (no grid). However, to make comparisons easier, we have applied the same scale as the one used in the second row. The second row, that is, (d), (e), (f) shows stable configurations for *afuzzy* $= 0.05$, *bfuzzy* $= 0.1$, and $pd/sd = 0.05$. In the third row, i.e., (g), (h), (i) there are configurations obtained for *afuzzy* $= 0.01$, *bfuzzy* $= 0.1$, and $pd/sd = 0.3$. The scale here is different, which results from applied LP parameter values. The values of f are calculated according to formula (11). (a) $f = 0.060$ (b) $f = 0.112$; (c) $f = 0.075$; (d) $f = 0.083$, Mean_truth(P1) = 1; Mean_truth(P3) = 0.99; (e) $f = 0.122$, Mean_truth(P1) = 1; Mean_truth(P3) = 1; (f) $f = 0.096$, Mean_truth(P1) = 1; Mean_truth(P3) = 0.98; (g) $f = 0.060$, Mean_truth(P1) = 0.4; Mean_truth(P3) = 0.92; (h) $f = 0.111$, Mean_truth(P1) = 0.19; Mean_truth(P3) = 0.85; (i) $f = 0.074$, Mean_truth(P1) = 0.28; Mean_truth(P3) = 0.88.

definitions for given patterns, (the *afuzzy* value from Fig. 12) gradually decreased, while the pd/sd value (Fig. 14) – increased. The configurations presented in the third row of Fig. 20 (the best of 10 trials) were obtained for *afuzzy* $= 0.01$, *bfuzzy* = 0.1, and $pd/sd = 0.3$.

As can be seen from the graphs presented in row 3, the resulting configurations are very similar in quality to those obtained using eigenvectors. Furthermore, the assessment of their function f was analogous and even slightly better for examples 2 and 3. However, obtaining a high mean truth value for the P1 pattern is not possible due to the distance definitions adopted in these simulations. The pd/sd and *afuzzy* parameters would generally result in the layout of objects over a larger area than in the second row presented in Fig. 20 and the lower Mean_truth(P1).

7. Discussion and possible applications

The simulation examples presented were designed for illustrative purposes of the proposed methodology. The analyzed models of qualitatively known optimal configurations (Figs. 8, 12 and 13) reproduced them in repeated simulations. For the bivariate evaluations t(l) (Fig. 13), they are even surprising and suggest that some analytical solutions exist and could be found for these cases. The properties of the configurations obtained in the migration models studied (Fig. 17, 19) are generally consistent with

Table 3
Drezner's examples for scattered plots.

Facility	Should be close to facility		
	Drezner1	Drezner2	Drezner3
1	8 10 15	5 6 10 12 17 18	7 8 9
2	6 14 15	6 7 12 13 14 16	4 6 8
3	11 18 19	8 9 15 16	10 13 16
4	6 10 13	9 11 15 17	2 15 17
5	10 16 18	1 11 17 18	9 18 19
6	2 4 14 17	1 2 10 12 13 19	2 5 11
7	9 13 19	2 13 14	1 12 14
8	1 15 16	3 14 16	1 2 18
9	7 12 17	3 4 12 15 16 17	1 10 11
10	1 4 5 16	1 6 18 19	3 9 12
11	3 13 18	4 5 17	6 9 19
12	9 14 17	1 2 6 9 16 17	7 10 16
13	4 7 11 19	2 6 7 19	3 15 17
14	2 6 12	2 7 8 16	7 17 18 19
15	1 2 8	3 4 9	4 13 16
16	5 8 10	2 3 8 9 12 14	3 12 15
17	6 9 12	1 4 5 9 11 12	4 13 14
18	3 5 11	1 5 10	5 8 14
19	3 7 13	6 10 13	5 11 14

the results from the Sakoda and Schelling models. The observed differences arise logically from the assumptions made in our approach. Analysis of Drezner's [7] scattered plot examples (Fig. 20) indicates that our LP concept gives much more flexibility in modeling this type of problem, which allows it to be used in specific practical implementations. For example, it is possible to include the dimensions of objects, which is impossible in the classical Drezner optimization method.

In general, the incorporation of knowledge-based LPs has expanded the modeling possibilities and moved beyond treating agents as individuals. As a result, the approach can be applied in completely different contexts, and it allows for modeling practical issues other than social group migration. For example, the freedom to define LPs makes it possible to obtain scatter plots for facility layout problems. In this area, it may be interesting to analyze the desired locations and neighborhoods of the collaborating human team members. The results of such modeling can be used, for example, to design their arrangements in open office spaces. Furthermore, scattered plots may facilitate the determination of the desired arrangement of greenfield-designed factory components, or production systems. In the latter cases, agents can be interpreted as *interacting* buildings and/or machines.

Importantly, relationships and variables are specified for each task using formulations defined by expert knowledge in terms similar to natural language. Overall, in any application problem, one has to create such linguistic patterns that reflect logical relationships and the desired state of the examined system in reality. Since often such a formal description by classic mathematical formulae may be difficult due to the information uncertainty, the fuzzy sets and linguistic patterns appear to be well fitted to this job. The determination of the appropriate patterns can be obtained, for instance, by finding a consensus between the knowledge of different experts within the given field or in concrete situations. For example, such a compromise for the arrangement of production machines within the factory layout may be expressed by a linguistic pattern of the following form: 'IF Transport_between_machines(i, j) IS FREQUENT THEN Distance_between_machines(i, j) IS SMALL'. Experts should define the fuzzy set membership functions for FREQUENT and SMALL, based on their knowledge, experience, and available data. The proposed linguistic expression corresponds to the logical economic requirement of arranging objects to minimize transportation costs.

The features described above and the flexibility of our approach also have some negative consequences. It provides a relatively large number of degrees of freedom in model construction and analysis. Therefore, the properties of this methodology require further detailed research, in particular on the sensitivity of the model to changes in various parameters, LP definitions, etc.

There are many possibilities to improve and extend the method proposed in this work. For example, by broadening the concept and its implementation to include additional evaluation criteria defined by LPs. These could, for example, involve simultaneous consideration of safety recommendations in the design of production halls, interaction between facilities, social or cultural preferences between groups of workers, or aesthetic evaluation of design solutions. Such a multi-criteria approach would allow modeling, analyzing, and searching for solutions in much more complex systems. As far as the implementation of the proposed methodology is concerned, the potential extension should allow movement parameters to be set for individual agents or their defined groups. An addition of this type would even more strongly increase the flexibility of the proposed approach in modeling practical issues from different areas. In the future, the corresponding computer implementations of our proposal should take into account the ability to independently and freely construct LPs and define linguistic variables.

The examples presented in this paper suggest that the LP-based approach may be an interesting perspective to analyze the dynamics of interconnected agents with different mutual attitudes in various contexts. A unique feature of this approach is the ability to define mutual relations and behavioral rules using expressions similar to those found in natural language. They model imprecisely defined behavioral rules for agents. The rules of multivalued logic and fuzzy sets applied to such modeling are a generalization of traditional bivalued logic and sets. Thus, our models can also operate on exact data (physical properties) and/or combine various types of data and relations. The implemented model of agent behavior, although evaluated and validated rather qualitatively, is promising. In all cases, a stable configuration of the agents was achieved relatively quickly.

The interaction rules in our approach are simple, the same for each agent, and the number of agents is significant. According to the classification proposed by Kliemt [59], this type of model belongs to the *thin* group. At the other extreme of the mentioned classification are *thick* models. This type of modeling consists in reproducing, as accurately as possible, the knowledge about characteristics and behavior of a comparatively small group of diverse agents as, for example, in [60,61]. In these works, the characteristics and rules of agent behavior were constructed based on multidisciplinary knowledge of consumer behavior, social psychology, marketing, and organizational culture. Since our LP-based proposal facilitates flexible design of patterns using multiple, natural language-like expressions, it allows one to easily take the knowledge of multiple experts from different fields and encapsulate it in a single approach. Therefore, the design of *thick* models seems to be another interesting challenge and a direction for further exploration of the possibilities offered by our approach.

8. Conclusions

In this research, we showed how the concept of LPs combined with expert knowledge can be used to model the dynamics of social groups. This approach belongs to the domain of agent-based modeling that involves migration.

In our proposal, linguistic phrases similar to natural language define ideal properties of the examined system. They are the basis for generating virtual forces that govern the movements of the individual agent. The development of agent behavior rules results from logical sentences and the methodology to determine their degree of truth. Such an approach allows us to construct flexible simulation models.

In this paper, we not only describe our idea in detail, but also illustrate its capabilities and properties by simple examples and a series of simulation experiments. They include problems of known solutions with stable final configurations that were analyzed. The analyses show that qualitative results of classical ideas from previous works can be obtained without original limitations such as moving on a grid. These models, as shown in the examples, also use the paradigm of inferring the behavior of the dynamics of the entire social system based on the interactions between its members.

We also validated our approach by applying and comparing it with a suboptimal scattered plot generation method proposed by Drezner. The simulations performed for classic problems showed the convergence of agent dynamic behavior in our method with the solutions provided by the Drezner approach.

As was broadly discussed in the previous section, the presented method can be potentially widely used in a variety of situations, and the proposed framework can be easily extended to model other types of complex systems.

CRediT authorship contribution statement

Jerzy Grobelny: Supervision, Conceptualization, Methodology, Software, Visualization, Investigation, Data curation, Writing – original draft, Writing – review & editing. **Rafał Michalski:** Conceptualization, Methodology, Software, Visualization, Investigation, Data curation, Writing – original draft, Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This research was partially financially supported by the Polish National Science Centre under Grant No. 2019/35/B/HS4/02892. The authors thank four anonymous reviewers for carefully reading and providing substantive feedback on earlier versions of the

```
Define input data:
    Links matrix L_{n×n}
    Patterns definitions and relationships of calculating t(l) and t(r)
    Determination of the (rectangular) area
    Arrange randomly n agents in the available area
    Determine maximal number of steps in simulation t and values of s and r
    count = 1,
    Repeat
        For i = 1 to n do
            Begin
                VF = 0
                  For j = 1 (and j <> i) to n do
                      Begin
                          If Distance(i, j) <= r then
                              Begin
                                  For each Pattern do
                                      Begin
                                          Determine pattern Truth(i, j)
                                          Determine vector VF(i, j)
                                          VF := VF + VF(i, j)
                                      End
                              End(If)
                          Move agent i according to VF adjusted by s/n
                      End
                  Calculate Mean_truth values for agent i (for all patterns)
            End
        Calculate and write Mean_truth values for all patterns and all agents
        count = count  + 1
  Until count = t + 1
```

manuscript. Their valuable comments and suggestions helped to make the paper significantly better.

Appendix

The proposed model pseudocode:

References

[1] J.M. Sakoda, The checkerboard model of social interaction, J. Math. Sociol. 1 (1) (1971) 119–132, http://dx.doi.org/10.1080/0022250X.1971.9989791.
[2] T.C. Schelling, Models of segregation, Am. Econ. Rev. 59 (2) (1969) 488–493.
[3] T.C. Schelling, Dynamic models of segregation, J. Math. Sociol. 1 (2) (1971) 143–186, http://dx.doi.org/10.1080/0022250X.1971.9989794.
[4] J. Grobelny, The fuzzy approach to facilities layout problems, Fuzzy Sets and Systems 23 (2) (1987) 175–190, http://dx.doi.org/10.1016/0165-0114(87)90057-1.
[5] J. Grobelny, The 'linguistic pattern' method for a workstation layout analysis, Int. J. Prod. Res. 26 (11) (1988) 1779–1798, http://dx.doi.org/10.1080/00207548808947991.
[6] F.S. Beltran, L. Salas, V. Quera, Spatial behavior in groups: an agent-based approach, J. Artif. Soc. Soc. Simul. 9 (3) (2006) 1–5.
[7] Z. Drezner, A heuristic procedure for the layout of a large number of facilities, Manage. Sci. 33 (7) (1987) 907–915, http://dx.doi.org/10.1287/mnsc.33.7.907.
[8] I.N. Lymperopoulos, G.D. Ioannou, Understanding and modeling the complex dynamics of the online social networks: a scalable conceptual approach, Evol. Syst. 7 (3) (2016) 207–232, http://dx.doi.org/10.1007/s12530-016-9145-9.
[9] R. Mercuur, V. Dignum, C.M. Jonker, Integrating social practice theory in agent-based models: A review of theories and agents, IEEE Trans. Comput. Soc. Syst. 7 (5) (2020) 1131–1145, http://dx.doi.org/10.1109/TCSS.2020.3007930.
[10] J. Castro, et al., A review of agent-based modeling of climate-energy policy, WIREs Clim. Chang. 11 (4) (2020) e647, http://dx.doi.org/10.1002/wcc.647.
[11] R. de A. Silva, R.T.V. Braga, Simulating systems-of-systems with agent-based modeling: A systematic literature review, IEEE Syst. J. 14 (3) (2020) 3609–3617, http://dx.doi.org/10.1109/JSYST.2020.2980896.
[12] A. Micolier, P. Loubet, F. Taillandier, G. Sonnemann, To what extent can agent-based modelling enhance a life cycle assessment? Answers based on a literature review, J. Cleaner Prod. 239 (2019) 118123, http://dx.doi.org/10.1016/j.jclepro.2019.118123.
[13] S.M. Reia, A.C. Amado, J.F. Fontanari, Agent-based models of collective intelligence, Phys. Life Rev. 31 (2019) 320–331, http://dx.doi.org/10.1016/j.plrev.2018.10.004.
[14] K. Rakić, M. Rosić, I. Boljat, A survey of agent-based modelling and simulation tools for educational purpose, Teh. Vjesn. 27 (3) (2020) 1014–1020, http://dx.doi.org/10.17559/TV-20190517110455.
[15] C. Santos, M.J.P. Neto, M. Neves, The influence of infographics in accessing information: Multidimensionality in visual representation and configuration of different media, in: Advances in Ergonomics in Design, 2019, pp. 497–508.
[16] P. Hansen, X. Liu, G.M. Morrison, Agent-based modelling and socio-technical energy transitions: A systematic literature review, Energy Res. Soc. Sci. 49 (2019) 41–52, http://dx.doi.org/10.1016/j.erss.2018.10.021.
[17] H.R. Nasrinpour, M.R. Friesen, R.D. McLeod, Agent based modelling and west nile virus: A survey, J. Med. Biol. Eng. 39 (2) (2019) 178–183, http://dx.doi.org/10.1007/s40846-018-0396-8.
[18] S. Flaxman, et al., Estimating the effects of non-pharmaceutical interventions on COVID-19 in europe, Nature 584 (7820) (2020) 257–261, http://dx.doi.org/10.1038/s41586-020-2405-7.
[19] L. Wynants, et al., Prediction models for diagnosis and prognosis of covid-19: systematic review and critical appraisal, BMJ 369 (2020) m1328, http://dx.doi.org/10.1136/bmj.m1328.
[20] M.J. North, A theoretical formalism for analyzing agent-based models, Complex Adapt Syst. Model 2 (1) (2014) 3, http://dx.doi.org/10.1186/2194-3206-2-3.
[21] R. Hegselmann, A. Flache, Understanding complex social dynamics, J. Artif. Soc. Soc. Simul. 1 (3) (2003) 1998, [Online]. Available: https://www.jasss.org/6/4/8.html. (Accessed 30 Oct. 2021).
[22] J.S. Klüver, Simulations of group dynamics with different models, J. Artif. Soc. Soc. Simul. 6 (4) (2003) [Online]. Available: https://www.jasss.org/6/4/8.html. (Accessed 30 Oct. 2021).
[23] J.L. Moreno, Who Shall Survive? a New Approach to the Problem of Human Interrelations, Nervous and Mental Disease Pub. Co. Washington, 1934.
[24] E.T. Hall, The Hidden Dimension, Doubleday, Garden City, N.Y., 1966.
[25] S. Hoogendoorn, P.H.L. Bovy, Simulation of pedestrian flows by optimal control and differential games, Optim. Control Appl. Meth. 24 (3) (2003) 153–172, http://dx.doi.org/10.1002/oca.727.
[26] D. Helbing, I. Farkas, T. Vicsek, Simulating dynamical features of escape panic, Nature 407 (6803) (2000) 487–490, http://dx.doi.org/10.1038/35035023.
[27] L. Gao, B. Durnota, Y. Ding, H. Dai, An agent-based simulation system for evaluating gridding urban management strategies, Knowl.-Based Syst. 26 (2012) 174–184, http://dx.doi.org/10.1016/j.knosys.2011.07.018.
[28] S. Sharma, K. Ogunlana, D. Scribner, J. Grynovicki, Modeling human behavior during emergency evacuation using intelligent agents: A multi-agent simulation approach, Inf. Syst. Front. 20 (4) (2018) 741–757, http://dx.doi.org/10.1007/s10796-017-9791-x.
[29] B. Yıldız, G. Çağdaş, Fuzzy logic in agent-based modeling of user movement in urban space: Definition and application to a case study of a square, Build. Environ. 169 (2020) 106597, http://dx.doi.org/10.1016/j.buildenv.2019.106597.
[30] I. Moya, M. Chica, J.L. Sáez-Lozano, Ó. Cordón, An agent-based model for understanding the influence of the 11-M terrorist attacks on the 2004 Spanish elections, Knowl.-Based Syst. 123 (2017) 200–216, http://dx.doi.org/10.1016/j.knosys.2017.02.015.
[31] H. Bin, D. Zhang, Cellular-automata based qualitative simulation for Non-profit group behavior, J. Artif. Soc. Soc. Simul. 10 (1) (2007) [Online]. Available: https://www.jasss.org/10/1/1.html. (Accessed 30 Oct. 2021).
[32] W. Yu, D. Helbing, Game theoretical interactions of moving agents, in: J. Kroc, P.M.A. Sloot, A.G. Hoekstra (Eds.), Simulating Complex Systems By Cellular Automata, Springer, Berlin, Heidelberg, 2010, pp. 219–239, http://dx.doi.org/10.1007/978-3-642-12203-3_10.
[33] D. Helbing, W. Yu, H. Rauhut, Self-organization and emergence in social systems: Modeling the coevolution of social environments and cooperative behavior, J. Math. Sociol. 35 (1–3) (2011) 177–208, http://dx.doi.org/10.1080/0022250X.2010.532258.
[34] A. Kowalska-Styczeń, K. Sznajd-Weron, Access to information in word of mouth marketing within a cellular automata model, Adv. Complex Syst. 15 (8) (2012) 1250080.
[35] T. Ma, Y. Nakamori, Agent-based simulation for kansei engineering: Testing a fuzzy linear quantification method in an artificial world, J. Syst. Sci. Syst. Eng. 16 (3) (2007) 308–322, http://dx.doi.org/10.1007/s11518-007-5055-5.
[36] L. Iandoli, E. Marchione, C. Ponsiglione, G. Zollo, Computing ambiguity in complex systems with fuzzy logic, Fuzzy Econ. Rev. XIV (2009) 67–90, http://dx.doi.org/10.25102/fer.2009.01.04.
[37] R.R. Yager, Families of OWA operators, Fuzzy Sets and Systems 59 (2) (1993) 125–148, http://dx.doi.org/10.1016/0165-0114(93)90194-M.
[38] S. Hassan, L. Garmendia, J. Pavon, Introducing uncertainty into social simulation: using fuzzy logic for agent-based modelling, Int. J. Reason. Based Intell. Syst. (2010) [Online]. Available: https://www.inderscienceonline.com/doi/abs/10.1504/IJRIS.2010.034907. (Accessed 18 Jan. 2022).
[39] S. Hassan, M. Salgado, J. Pavón, Friendship dynamics: Modelling social relationships through a fuzzy agent-based simulation, Discrete Dyn. Nat. Soc. 2011 (2011) e765640, http://dx.doi.org/10.1155/2011/765640.
[40] S. Anand, I. Sharma, R. Goyal, Extended understanding of dyadic friendship using fuzzy measures: a simulation approach, IEEE Access 5 (2017) 21179–21192, http://dx.doi.org/10.1109/ACCESS.2017.2758395.
[41] J. Martínez-Miranda, J. Pavón, Modeling the influence of trust on work team performance, Simulation 88 (4) (2012) 408–436, http://dx.doi.org/10.1177/0037549711404714.
[42] M. Raoufi, A. Robinson Fayek, Fuzzy agent-based modeling of construction crew motivation and performance, J. Comput. Civ. Eng. 32 (5) (2018) 04018035, http://dx.doi.org/10.1061/(ASCE)CP.1943-5487.0000777.
[43] N.S. Kedir, M. Raoufi, A.R. Fayek, Fuzzy agent-based multicriteria decision-making model for analyzing construction crew performance, J. Manage. Eng. 36 (5) (2020) 04020053, http://dx.doi.org/10.1061/(ASCE)ME.1943-5479.0000815.
[44] M. Raoufi, A.R. Fayek, Hybrid fuzzy Monte Carlo agent-based modeling of workforce motivation and performance in construction, Constr. Innov. 21 (3) (2021) 398–416, http://dx.doi.org/10.1108/CI-03-2020-0045.
[45] S. Mei, et al., Individual decision making can drive epidemics: A fuzzy cognitive map study, IEEE Trans. Fuzzy Syst. 22 (2) (2014) 264–273, http://dx.doi.org/10.1109/TFUZZ.2013.2251638.
[46] P.J. Giabbanelli, S.A. Gray, P. Aminpour, Combining fuzzy cognitive maps with agent-based modeling: Frameworks and pitfalls of a powerful hybrid modeling approach to understand human–environment interactions, Environ. Model. Softw. 95 (2017) 320–325, http://dx.doi.org/10.1016/j.envsoft.2017.06.040.
[47] S. Mehryar, R. Sliuzas, N. Schwarz, A. Sharifi, M. van Maarseveen, From individual fuzzy cognitive maps to agent based models: Modeling multi-factorial and multi-stakeholder decision-making for water scarcity, J. Environ. Manag. 250 (2019) 109482, http://dx.doi.org/10.1016/j.jenvman.2019.109482.

[48] J. Giráldez-Cru, M. Chica, O. Cordón, F. Herrera, Modeling agent-based consumers decision-making with 2-tuple fuzzy linguistic perceptions, Int. J. Intell. Syst. 35 (2) (2020) 283–299, http://dx.doi.org/10.1002/int.22211.

[49] J. Giráldez-Cru, M. Chica, O. Cordón, A framework of opinion dynamics using fuzzy linguistic 2-tuples, Knowl.-Based Syst. 233 (2021) 107559, http://dx.doi.org/10.1016/j.knosys.2021.107559.

[50] A.E. Badillo-Márquez, A.A. Aguilar-Lasserre, M.A. Miranda-Ackerman, O.O. Sandoval-González, D. Villanueva-Vásquez, R. Posada-Gómez, An agent-based model-driven decision support system for assessment of agricultural vulnerability of sugarcane facing climatic change, Mathematics 9 (23) (2021) http://dx.doi.org/10.3390/math9233061, (23).

[51] E.H. Mamdani, S. Assilian, An experiment in linguistic synthesis with a fuzzy logic controller, Int. J. Man-Mach. Stud. 7 (1) (1975) 1–13, http://dx.doi.org/10.1016/S0020-7373(75)80002-2.

[52] J. Grobelny, On one possible 'fuzzy' approach to facilities layout problems, Int. J. Prod. Res. 25 (8) (1987) 1123.

[53] A.D. Raoot, A. Rakshit, A 'linguistic pattern' approach for multiple criteria facility layout problems, Int. J. Prod. Res. 31 (1) (1993) 203–222, http://dx.doi.org/10.1080/00207549308956721.

[54] J. Grobelny, R. Michalski, A novel version of simulated annealing based on linguistic patterns for solving facility layout problems, Knowl.-Based Syst. 124 (2017) 55–69, http://dx.doi.org/10.1016/j.knosys.2017.03.001.

[55] J. Grobelny, Fuzzy-based linguistic patterns as a tool for the flexible assessment of a priority vector obtained by pairwise comparisons, Fuzzy Sets and Systems 296 (2016) 1–20, http://dx.doi.org/10.1016/j.fss.2015.05.012.

[56] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980.

[57] L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1) (1978) 3–28, http://dx.doi.org/10.1016/0165-0114(78)90029-5.

[58] L.A. Zadeh, PRUF—a meaning representation language for natural languages, Int. J. Man-Mach. Stud. 10 (4) (1978) 395–460, http://dx.doi.org/10.1016/S0020-7373(78)80003-0.

[59] H. Kliemt, Simulation and rational practice, in: R. Hegselmann, U. Mueller, K.G. Troitzsch (Eds.), Modelling and Simulation in the Social Sciences from the Philosophy of Science Point of View, Springer Netherlands, Dordrecht, 1996, pp. 13–27, http://dx.doi.org/10.1007/978-94-015-8686-3_2.

[60] T.J. Norman, et al., Agent-based formation of virtual organisations, Knowl.-Based Syst. 17 (2–4) (2004) 103–111, http://dx.doi.org/10.1016/j.knosys.2004.03.005.

[61] O. Roozmand, N. Ghasem-Aghaee, G.J. Hofstede, M.A. Nematbakhsh, A. Baraani, T. Verwaart, Agent-based modeling of consumer decision making process based on power distance and personality, Knowl.-Based Syst. 24 (7) (2011) 1075–1095, http://dx.doi.org/10.1016/j.knosys.2011.05.001.